issue brief

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Duration Basics

Introduction

Duration is a term used by fixed-income investors, financial advisors, and investment advisors. It is an important measure for investors to consider, as bonds with higher durations (given equal credit, inflation and reinvestment risk) may have greater price volatility than bonds with lower durations. It is an important tool in structuring and managing a fixed-income portfolio based on selected investment objectives.

Investment theory tells us that the value of a fixed-income investment is the sum of all of its cash flows discounted at an interest rate that reflects the inherent investment risk. In addition, due to the time value of money, it assumes that cash flows returned earlier are worth more than cash flows returned later. In its most basic form, duration measures the weighted average of the present value of the cash flows of a fixed-income investment.

All of the components of a bond—price, coupon, maturity, and interest rates—are used in the calculation of its duration. Although a bond's price is dependent on many variables apart from duration, duration can be used to determine how the bond's price may react to changes in interest rates.

This issue brief will provide the following information:

- A basic overview of bond math and the components of a bond that will affect its volatility.
- The different types of duration and how they are calculated.
- Why duration is an important measure when comparing individual bonds and constructing bond portfolios.
- An explanation of the concept of convexity and how it is used in conjunction with the duration measure.

Basic Bond Math and Risk Measurement

The price of a bond, or any fixed-income investment, is determined by summing the cash flows discounted by a rate of return. The rate of return can change at any time period and will be reflected in the calculation of an investment's market price.

The price of a bond can be calculated using the formula found in Figure 1.

Figure 1 - Bond Pricing Formula

$$\text{Bond Price} = \sum_{t=1}^{N} \underbrace{\frac{CPN_t}{\{1 + YTM_t\}^t}}_{\text{Coupon Cash Flows}} + \underbrace{\frac{P_n}{\{1 + YTM_n\}^n}}_{\text{Principal Repayment}}$$

Definitions:

CPN	**= coupon payment**
P	**= principal payment**
YTM	**= yield to maturity**
n	**= number of compounding periods**
t	**= time period**

In reviewing this basic formula, it can be seen that the sensitivity of a bond's value to changing interest rates depends on both the length of time to maturity and on the pattern of cash flows provided by the bond.

As shown, there are many variables associated with pricing a bond. Changes in each of these variables, taken separately and in combination, can have a significant effect on price. The following basic principles are universal for bonds.

In general:

Changes in the value of a bond are inversely related to changes in the rate of return.

The higher the rate of return (i.e., yield to maturity (YTM)), the lower the bond value.

Long-term bonds have greater interest rate risk than short-term bonds.

There is a greater probability that interest rates will rise (increase YTM) and thus negatively affect a bond's market price, within a longer time period than within a shorter period.

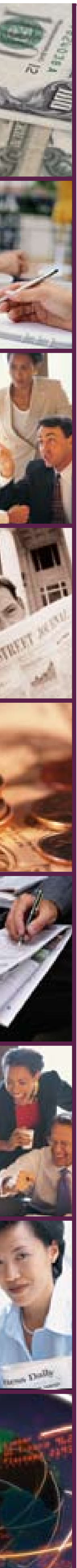
Low coupon bonds have greater interest rate sensitivity than high coupon bonds.

In other words, the more cash flow received in the short-term (because of a higher coupon), the faster the cost of the bond will be recovered. The same is true of higher yields. Again, the more a bond yields in today's dollars, the faster the investor will recover its cost.

What is Duration?

Duration can be used as a measure of risk in bond investing. While it comes in many forms, the ones most commonly used by public fund investors include the following:

Macaulay Duration. Developed in 1938 by Frederic Macaulay, this form of duration measures the number of years required to recover the true cost of a bond, considering the present value of all coupon and principal payments received in the future. Thus, it is the only type of duration



quoted in "years." Interest rates are assumed to be continuously compounded.

Modified Duration. This measure expands or modifies Macaulay duration to measure the responsiveness of a bond's price to interest rate changes. It is defined as the percentage change in price for a 100 basis point change in interest rates. The formula assumes that the cash flows of the bond do not change as interest rates change (which is not the case for most callable bonds).

Effective Duration. Effective duration (sometimes called option-adjusted duration) further refines the modified duration calculation and is particularly useful when a portfolio contains callable securities. Effective duration requires the use of a complex model for pricing bonds that adjusts the price of the bond to reflect changes in the value of the bond's "embedded options" (e.g., call options or a sinking fund schedule) based on the probability that the option will be exercised. Effective duration incorporates a bond's yield, coupon, final maturity and call features into one number that indicates how price-sensitive a bond or portfolio is to changes in interest rates.

For example, the price of a bond with an effective duration of two years will rise (fall) two percent for every one percent decrease (increase) in yield, The longer the duration, the more sensitive a bond is to changes in interest rates.

The type of duration measure used will depend upon several factors including the type of investments being analyzed (e.g., bullet securities versus callable securities) and the preference for calculating the measure using generally available in-house tools (which can be used to calculate Macaulay or modified duration) versus purchasing or relying on software that will create a simulation model of various interest rate scenarios for calculating effective duration.

Macaulay and Modified Duration Formulas

The following section will provide the calculations for determining the value of Macaulay and modified duration. The calculation for effective duration is complicated and involves averaging the duration under a simulation model of many possible interest rate scenarios in the future; thus, no example for this calculation appears below.[1]

Macaulay Duration Formula

Figure 2 – Macaulay Duration

$$\text{Macaulay Duration} = \sum_{t=1}^{n} \frac{(PV)\,(CF_t) \times t}{\text{Market Price of Bond}}$$

Definitions:

$(PV)(CF_t)$ = present value of coupon at period t

t = time to each cash flow (in years)

n = number of periods to maturity

[1] There are computer simulation programs available to investors that calculate effective duration.

Consider the following example:

Using the bond pricing formula in Figure 1, if interest rates were at 7 percent, a 3-year bond with a 10 percent coupon paid annually would sell for:

Market Price $= \$100/(1.07)^1 + \$100/(1.07)^2 + \$1100/(1.07)^3$
$= \$93.46 + \$87.34 + \$897.93$
$= \mathbf{\$1,078.73}$

Using the Macaulay duration formula in Figure 2, duration can be calculated as:

Macaulay Duration $= (1 \times \$93.46 / \$1,078.73) + (2 \times \$87.34 / \$1,078.73) + (3 \times \$897.93 / \$1,078.73)$
$= \mathbf{2.7458}$

Result: It takes **2.7458 years** to recover the true cost of the bond.

Modified Duration Formula

As shown in Figure 3, modified duration is an extension of Macaulay duration because it takes into account interest rate movements by including the frequency of coupon payments per year.

Figure 3 – Modified Duration

$$\text{Modified Duration} = \frac{\text{Macaulay Duration}}{1 + \dfrac{\text{Yield to maturity}}{\text{Number of coupon periods per year}}}$$

Using the previous example, yield to maturity is assumed to be 7 percent, there is 1 coupon period per year and the Macaulay duration is 2.7458. Solving for modified duration:

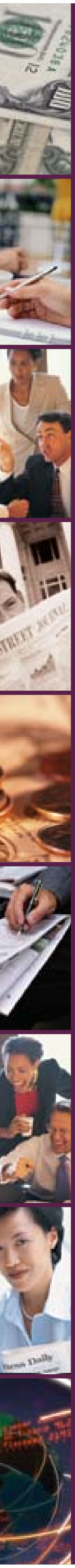
Modified Duration $= 2.7458 / 1+(.07/1)$
$= 2.7458 / 1.07$
$= \mathbf{2.566}$

Result: For every 1 percent change in market interest rates, the market value of the bond will move inversely by **2.566 percent**.

Principles of Duration

As used in the equations in Figures 1 through 3 above, **coupon rate** (which determines the size of the periodic cash flow), **interest rates** (which determines the present value of the periodic cash flow), and **maturity** (which weights each cash flow) all contribute to the duration measures.

As maturity increases, duration increases and the bond's price becomes more sensitive to interest rate changes.



A decrease in maturity decreases duration and renders the bond less sensitive to changes in market yield. Therefore, duration varies directly with maturity.

As the bond coupon increases, its duration decreases and the bond becomes less sensitive to interest rate changes.

Increases in coupon rates raise the present value of each periodic cash flow and therefore the market price. This higher market price lowers the duration.

As interest rates increase, duration decreases and the bond becomes less sensitive to further rate changes.

As interest rates increase, all of the net present values of the future cash flows decline as their discount factors increase, but the cash flows that are farthest away will show the largest proportional decrease. So the early cash flows will have a greater weight relative to later cash flows. As yields decline, the opposite will occur.

Implications

- Duration allows bonds of different maturities and coupon rates to be directly compared.
- The higher the duration, the higher the risk of price changes as interest rates change.
- Constructing a bond portfolio based on weighted average duration provides the ability to determine value changes based on forecasted changes in interest rates.

Portfolio Duration

Duration is important to bond portfolio managers. The duration of a portfolio is the weighted average duration of all the bonds in the portfolio weighted by their dollar values (see Figure 4 for an example).

Figure 4 – Weighted Average Portfolio Duration

Bond	Market Value	Portfolio Weight	Duration	Weighted Duration
A	$100,000	.10	4	.4
B	$200,000	.40	7	2.8
C	$300,000	.30	6	1.8
D	$400,000	.20	2	.4
Total	**$1,000,000**	**1.00**		**5.4**

Managing Market Risk in Portfolios

Treasury managers may be able to modify interest rate risk by changing the duration of the portfolio. Figure 5 provides a simplified example of a $1,000,000 portfolio's gain or loss in market value based on changes to interest rates and/or the portfolio's modified duration.

Figure 5 – Gain/Loss of Market Value Matrix

$ (000)	Portfolio Modified Duration			
Rate Change	**1.0**	**3.0**	**5.0**	**6.0**
+25bp	$ 2.5	$ 7.5	$ 12.5	$ 15.0
+50bp	$ 5.0	$ 15.0	$ 25.0	$ 30.0
+100bp	$ 10.0	$ 30.0	$ 50.0	$ 60.0
+300bp	$ 30.0	$ 90.0	$ 150.0	$ 180.0
+500bp	$ 50.0	$ 150.0	$ 250.0	$ 300.0

Portfolio duration strategies may include reducing duration by adding shorter maturities or higher coupon bonds. They may increase duration by extending the maturities, or including lower-coupon bonds to the portfolio.

Each of these strategies can be employed based on the manager's propensity for active or passive investment management. If a treasury manager employs a passive management strategy, for example, targeting returns to a benchmark index, he or she may construct the portfolio to match the duration of the benchmark index. By contrast, an active strategy using benchmarks may include increasing the portfolio's duration to 105 percent of the benchmark during periods of falling rates, while reducing the duration to 95 percent of the benchmark during periods of rising rates.

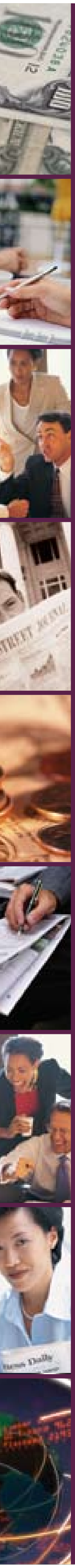
Portfolio Immunization Strategies

Another passive strategy, called "portfolio immunization," tries to protect the expected yield of a portfolio by acquiring securities whose duration equals the length of the investor's planned holding. This "duration matching" strategy attempts to manage the portfolio so that changes in interest rates will affect both price and reinvestment at the same rate, keeping the portfolio's rate of return constant.

Other portfolio immunization strategies not specifically associated with duration include bullet portfolio strategies, where maturities are centered at a single point on the yield curve; barbell portfolio strategies that concentrate maturities at two extreme points on the yield curve, with one maturity shorter and the other longer; and laddered portfolio strategies that focus on investments with staggered maturities allowing the reinvestment of principal from maturing lower-yield, shorter maturity bonds into new higher-yield, longer maturity bonds.

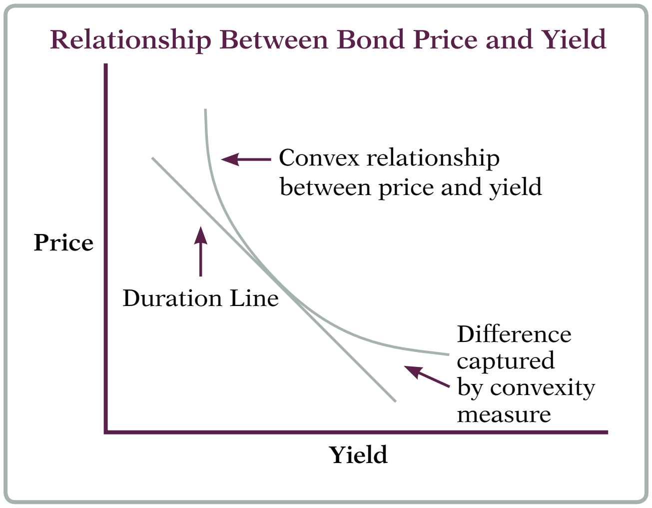
Convexity

One of the limitations of duration as a measure of interest rate/price sensitivity is that it is a linear measure. That is, it assumes that for a certain percentage change in interest rates that an equal percentage change in price will occur. However, as interest rates change, the price of a bond is not likely to change linearly, but instead would change over some curved, or "convex", function of interest rates.



For any given bond, a graph of the relationship between price and yield is convex. This means that the graph forms a curve rather than a straight line (see Figure 6).

Figure 6 – Convexity



The more convex the relationship the more inaccurate duration is as a measure of the interest rate sensitivity.

The convexity of a bond is a measure of the curvature of its price/yield relationship. The degree to which the graph is curved shows how much a bond's yield changes in response to a change in price.

Used in conjunction with duration, convexity provides a more accurate approximation of the percentage price change resulting from a specified change in a bond's yield than using duration alone. In addition to improving the estimate of a bond's price changes to changes in interest rates, convexity can also be used to compare bonds with the same duration. For example, two bonds may have the same duration but different convexity values. They may experience different price changes when there are extraordinary changes in interest rates. For example, if bond A has a higher convexity than bond B, its price would fall less during rising interest rates and appreciate more during falling interest rates as compared to bond B.

Duration and convexity are important measurement tools for use in valuation and portfolio management strategies. As such, they are an integral part of the financial services landscape. Duration and convexity functions are available in numerous financial management software packages and through Microsoft Excel. Bloomberg L.P. also includes the measures as a standard component of their bond presentation screens.

Conclusion

Duration is an important concept and tool available to all treasury managers who are responsible for managing a fixed-income portfolio.

Treasury managers may use duration to develop investment strategies that maximize returns while maintaining appropriate risk levels in a changing interest rate environment.

As with most financial management tools, duration does have certain limitations. A bond's price is dependent on many variables apart from the duration calculation and rarely correlates perfectly with the duration number.

With rates not moving in parallel shifts and the yield curve constantly changing, duration can be used to determine how the bond's price "may" react as opposed to "will" react. Nevertheless, it is an important tool available to treasury managers in the administration of their fixed-income portfolios.

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