CALIFORNIA DEBT AND INVESTMENT ADVISORY COMMISSION

# *Fundamentals of Debt Financing*
# *Basics of Bond Math*

***Presented by***


***Michelle Issa***
***Managing Director***
***(213) 229-3435***
***michelle.issa@bankofamerica.com***

***Karin E. Baker***
***Vice President***
***(213) 229-3439***
***karin.e.baker@bankofamerica.com***


*May 6, 2004*

Banc of America Securities

## Objective

Utilize a “real life” government example to teach you some bond math as well as how to “read” the debt service schedules that underwriters and financial advisors provide in the course of doing a bond financing.

# The Project

- The City of Pasadena has decided to capitalize on its worldwide reputation as home of the Tournament of Roses and the “Little Old Lady”
- The City has secured property on Colorado Boulevard to construct the museum to house:
  - Memorabilia from 115 years of the Rose Parade
  - History of the Rose Bowl, including video footage of key events; and
  - A wing honoring the Little Old Lady to include:
    - Her red Super Stock Dodge
    - A map of her typical route as she zoomed around Pasadena
    - The original album presenting her story signed by Jan and Dean; and
    - The first cut released by the Beach Boys signed by three out of four of the Beach Boys
- The City can finance its museum through a lease revenue bond financing

# Sample Bond Pricing Schedule

- The project cost is $42 million plus related reserves, capitalized interest and costs of issuance for a total bond financing of $50 million
- The City wants to finance the project over 20 years with level debt service
- Sample pricing schedule assumes all bonds are priced at par (or 100%)

| Maturity Date | Amount | Coupon | Yield | Price |
|---|---|---|---|---|
| 5/1/2007 | $ 1,895,000.00 | 2.050% | 2.050% | 100 |
| 5/1/2008 | $ 1,935,000.00 | 2.750% | 2.750% | 100 |
| 5/1/2009 | $ 1,985,000.00 | 3.250% | 3.250% | 100 |
| 5/1/2010 | $ 2,050,000.00 | 3.750% | 3.750% | 100 |
| 5/1/2011 | $ 2,125,000.00 | 4.250% | 4.250% | 100 |
| 5/1/2012 | $ 2,215,000.00 | 4.450% | 4.450% | 100 |
| 5/1/2013 | $ 2,315,000.00 | 4.600% | 4.600% | 100 |
| 5/1/2014 | $ 2,420,000.00 | 4.750% | 4.750% | 100 |
| 5/1/2024 | $ 33,060,000.00 | 5.750% | 5.750% | 100 |
| **Total** | **$ 50,000,000.00** | | | |

# Present Value Formula

- The basics of bond math is the Time Value of money as calculated with the Present Value and Future Value formulas
- The practical application of the PV formula is to price a bond. In our first example, the price of all bonds is 100 or par

**Present Value**

**Future Value**

$$PV = \frac{FV}{\left(1 + \frac{i}{n}\right)^{t}}$$

**Time (or period)**

*Municipal Convention:* $\frac{\frac{30}{360}\text{Days}}{180}$

**Compounding Periods Per Year**

*Municipal Convention: 2*

**Interest Rate (as decimal)**

0.0375 = 3.75%

# Time Value of Money

$$PV = \frac{FV}{\left(1+\frac{i}{n}\right)^{t}}$$

i = 3.50%
n = 1
FV = 1,000

- In this example, the Present Value is the amount that would have to be invested today at 3.5% to have $1,000 at the end of each of 10 periods

$PV_1$=1,000.00/(1+.035)$^1$
$PV_2$=966.18/(1+.035)$^1$ or =1,000.00/(1+.035)$^2$
$PV_3$=933.51/(1+.035)$^1$ or =1,000.00/(1+.035)$^3$

and so on…

| Period | Present Value |
|---|---|
| 1 | $ 966.18 |
| 2 | $ 933.51 |
| 3 | $ 901.94 |
| 4 | $ 871.44 |
| 5 | $ 841.97 |
| 6 | $ 813.50 |
| 7 | $ 785.99 |
| 8 | $ 759.41 |
| 9 | $ 733.73 |
| 10 | $ 708.92 |

# Time Value of Money

$$FV = PV * \left(1 + \frac{i}{n}\right)^t$$

i = 3.50%
n = 1
FV = 1,000

- In this example, Future Value is the amount that $1,000 becomes if it is invested at 3.5% at the end of each of 10 periods

$FV_1 = 1{,}000.00*(1+.035)^1$

$FV_2 = 1{,}035.00*(1+.035)^1$ or $= 1{,}000.00*(1+.035)^2$

$FV_3 = 1{,}071.23*(1+.035)^1$ or $= 1{,}000.00*(1+.035)^3$

and so on…

| Period | Future Value |
|---|---|
| 1 | $ 1,035.00 |
| 2 | $ 1,071.23 |
| 3 | $ 1,108.72 |
| 4 | $ 1,147.52 |
| 5 | $ 1,187.69 |
| 6 | $ 1,229.26 |
| 7 | $ 1,272.28 |
| 8 | $ 1,316.81 |
| 9 | $ 1,362.90 |
| 10 | $ 1,410.60 |

# Day Counts

- 30/360 Day Count
  - Municipal market convention for long-term bonds
  - Assumes 30 days in each month; 360 days in each year
  - The MSRB 30/360 Day Count formula:

$$\text{Number of Days} = (Y2 - Y1)360 + (M2 - M1)30 + (D2 - D1)$$

- "M1" is Month of Beginning Computation Date
- "D1" is Day of Beginning Computation Date
  - If "D1" = "31", change "D1" to "30"
- "Y1" is the Year of Beginning Computation Date
- "M2" is Month Ending Computation Date
- "D2" is Day Ending Computation Date
  - If "D2" = "31", and "D1" = "30" or "31", change "D2" to "30"
- "Y2" is Year of Ending Computation Date

- Other Day Count Methods: Actual/Actual, Actual/360, and Actual/365

# Day Counts

- 30/360 Day Count – Example

$$\text{Number of Days} = (Y2 - Y1)360 + (M2 - M1)30 + (D2 - D1)$$

- Date 1 = May 6, 2004
- Date 2 = November 1, 2007

Y1= 2004
Y2= 2007
M1= 5
M2= 11
D1= 6
D2= 1

$$\begin{aligned}\text{Number of Days} &= (2007 - 2004)360 + (11 - 5)30 + (1 - 6)\\ &= (3)360 + (6)30 + (-5)\\ &= 1{,}080 + 180 - 5\\ &= 1{,}255\end{aligned}$$

# Bond Pricing Formula

- Bond Price ~ present value of debt service
- Prices are truncated to three decimals
- MSRB pricing formula for more than six months to redemption

$$P = \left[\frac{RV}{\left(1+\frac{Y}{2}\right)^{\left(N-1+\frac{E-A}{E}\right)}}\right] + \left[\sum_{K=1}^{N}\frac{100*\frac{R}{2}}{\left(1+\frac{Y}{2}\right)^{\left(K-1+\frac{E-A}{E}\right)}}\right] - \left[100*\frac{A}{B}*R\right]$$

Present Value of Par Amount from Maturity Date to Settlement Date

Present Value of Interest from Payment Dates to Settlement Date

Interest Accrued from Dated Date to Settlement Date

| | |
|---|---|
| A= | 30/360 days from dated date to settlement date |
| B= | Days in the year: 360 |
| E= | Days in the semi-annual period: 180 |
| N= | Interest payments between settlement date and redemption date |
| P= | Dollar price |
| R= | Annual coupon (as decimal) |
| RV= | Redemption value |
| Y= | Yield (as decimal) |

# Bond Pricing Formula

- Example: Bond Priced at Par (Coupon = Yield)

$$\text{Price} = \frac{100}{\left(1+\frac{.015}{2}\right)^{\left(3-1+\frac{180-0}{180}\right)}} + \frac{100*\frac{.015}{2}}{\left(1+\frac{.015}{2}\right)^{\left(1-1+\frac{180-0}{180}\right)}} + \frac{100*\frac{.015}{2}}{\left(1+\frac{.015}{2}\right)^{\left(2-1+\frac{180-0}{180}\right)}} + \frac{100*\frac{.015}{2}}{\left(1+\frac{.015}{2}\right)^{\left(3-1+\frac{180-0}{180}\right)}} - 100*\frac{0}{360}*.015 =$$

| | |
|---|---|
| A= | 0 |
| B= | 360 |
| C= | 180 |
| N= | 3 |
| R= | 1.5% |
| RV= | 100 |
| Y= | 1.5% |

| | |
|---|---|
| Dated Date: | May 1, 2004 |
| Settlement Date: | May 1, 2004 |
| Maturity Date: | November 1, 2005 |
| Coupon: | 1.50% |
| Yield: | 1.50% |

Price = 97.7833 + (0.7444+0.7389+0.7334) – 0 = 100

Present Value of Par Amount from Maturity Date to Settlement Date

Present Value of Interest from Payment Dates to Settlement Date

Interest Accrued from Dated Date to Settlement Date

| | |
|---|---|
| A= | 30/360 days from dated date to settlement date |
| B= | Days in the year: 360 |
| E= | Days in the semi-annual period: 180 |
| N= | Interest payments between settlement date and redemption date |
| P= | Dollar price |
| R= | Annual coupon (as decimal) |
| RV= | Redemption value |
| Y= | Yield (as decimal) |

# Bond Pricing Formula

- Example: Bond Priced at a Premium (Coupon > Yield)

$$\text{Price} = \frac{100}{\left(1+\frac{.015}{2}\right)^{\left(3-1+\frac{180-0}{180}\right)}} + \frac{100 * \frac{.020}{2}}{\left(1+\frac{.015}{2}\right)^{\left(1-1+\frac{180-0}{180}\right)}} + \frac{100 * \frac{.020}{2}}{\left(1+\frac{.015}{2}\right)^{\left(2-1+\frac{180-0}{180}\right)}} + \frac{100 * \frac{.020}{2}}{\left(1+\frac{.015}{2}\right)^{\left(3-1+\frac{180-0}{180}\right)}} - 100 * \frac{0}{360} * .020 =$$

| | |
|---|---|
| A= | 0 |
| B= | 360 |
| C= | 180 |
| N= | 3 |
| R= | 2.00% |
| RV= | 100 |
| Y= | 1.50% |

| | |
|---|---|
| Dated Date: | May 1, 2004 |
| Settlement Date: | May 1, 2004 |
| Maturity Date: | November 1, 2005 |
| Coupon: | 2.00% |
| Yield: | 1.50% |

Price = 97.7833 + (0.9926+0.9852+0.9778) – 0 = 100.7389 = 100.738

Present Value of Par Amount from Maturity Date to Settlement Date

Present Value of Interest from Payment Dates to Settlement Date

Interest Accrued from Dated Date to Settlement Date

| | |
|---|---|
| A= | 30/360 days from dated date to settlement date |
| B= | Days in the year: 360 |
| E= | Days in the semi-annual period: 180 |
| N= | Interest payments between settlement date and redemption date |
| P= | Dollar price |
| R= | Annual coupon (as decimal) |
| RV= | Redemption value |
| Y= | Yield (as decimal) |

# Bond Amortization

- The amount of bonds that can be amortized annually depends on:
  - Revenue constraints
  - Debt service constraints
  - Bond coupons
- Once the constraints are known, a principal amortization schedule can be constructed

When sizing with revenue constraints, round down to nearest $5,000

## Bottom-up Approach:

| Year | Revenue/Debt Service Constraint | Coupon | Principal | Interest | Debt Service |
|---|---|---|---|---|---|
| 1 | 100,000.00 | 3.55% | $\frac{100{,}000.00 - 6{,}847.50}{(1 + .0355)} = 89{,}958.96 = 85{,}000.00$ | 3,562.50+3,285.00+3,017.50 | 94,865.00 |
| 2 | 100,000.00 | 3.65% | $\frac{100{,}000.00 - 3{,}562.50}{(1 + .0365)} = 93{,}041.49 = 90{,}000.00$ | 3,562.50+3,285.00 | 96,847.50 |
| 3 | 100,000.00 | 3.75% | $\frac{100{,}000.00}{(1 + .0375)} = 96{,}385.54 = 95{,}000.00$ | 3,562.50 | 98,562.50 |
| Maximum Principal: 270,000 +Interest 20,275 = Total Debt Service 290,275 | | | | | |

# Bond Amortization

- The pattern of debt service depends upon legal structure and/or issuer preferences

| | Level Debt Service | | | Level Principal | | | Deferred Debt Service | | |
|---|---|---|---|---|---|---|---|---|---|
| | *Principal* | *Coupon* | *Debt Service* | *Principal* | *Coupon* | *Debt Service* | *Principal* | *Coupon* | *Debt Service* |
| 6/1/05 | 31,980,000 | 3.55% | 35,632,780 | 33,330,000 | 3.55% | 36,980,005 | 6,820,000 | 3.55% | 10,510,795 |
| 6/1/06 | 33,260,000 | 3.65% | 35,777,490 | 33,335,000 | 3.65% | 35,801,790 | 45,565,000 | 3.65% | 49,013,685 |
| 6/1/07 | 34,760,000 | 3.75% | 36,063,500 | 33,335,000 | 3.75% | 34,585,063 | 47,615,000 | 3.75% | 49,400,563 |
| **TOTALS** | **100,000,000** | | **107,473,770** | **100,000,000** | | **107,366,858** | **100,000,000** | | **108,925,043** |

**Lease Secured Bonds**

- Required by State law
- Lengthens payout period
- Higher average life

**GO Bonds**

- Positive impact on rapidity of payout
- Offsets deferral required for lease financings

**Revenue Bonds**

- Specific situations

# Example: New Money Issue for the Pasadena Historical Museum Project

- Assumptions:

  Dated Date: May 6, 2004

  Delivery/Settlement Date: May 6, 2004

  First Interest Payment Date: Nov. 1, 2004

  First Principal Date: May 1, 2007

  Final Maturity: May 1, 2024

  Underwriter's Discount: $5.00/Bond

  Costs of Issuance: $200,000

  Project Fund: 12 even monthly draws invested at 2.5% net funded

  Capitalized Interest: 18 months, net funded, invested at 2.5%

  Debt Service Reserve Fund: Gross funded invested at arbitrage yield

  Optional Redemption Date: May 1, 2014

# Example: New Money Issue for the Pasadena Historical Museum Project

- Bond Financing Reports
  - Sources and Uses
  - Bond Summary Statistics
  - Bond Pricing
  - Bond Debt Service
  - Net Debt Service
  - Project Fund
  - Reserve Funds
    - Capitalized Interest
    - Debt Service Reserve Fund

# Sources and Uses

- Sources
  - Bond Proceeds
  - Other potential "sources"
    - Bond premium
    - Issuer/borrower contribution
    - Existing bond funds (for refunding issues)

| SOURCES | |
|---|---|
| Bond Proceeds | |
| Par Amount | $ 50,000,000.00 |
| Total | $ 50,000,000.00 |

# Sources and Uses

| USES | |
|---|---|
| Project Fund Deposits | |
| Project Fund | $ 41,766,602.69 |
| Other Fund Deposits | |
| Capitalized Interest Fund | $ 3,342,527.31 |
| Debt Service Reserve Fund | $ 4,440,870.00 |
| Delivery Date Expenses | |
| Cost of Issuance | $ 200,000.00 |
| Underwriter's Discount | $ 250,000.00 |
| Total | $ 50,000,000.00 |

# Sources and Uses

- Uses
  - Project Fund – Net funded for 12 even monthly draws
- Other Fund Deposits
  - Capitalized Interest Fund – Calculated to pay interest on bonds while project is being built prior to the City's ability to pay debt service
  - Debt Service Reserve Fund – Based on tax law, calculated as the lesser of:

    (i) 10% of Par Amount; (ii) 125% of Average Annual Debt Service; or (iii) Maximum Annual Debt Service

In our example:

| | |
|---|---|
| 10% of Par Amount ($50,000,000) = | $ 5,000,000.00 |
| 125% of Average Annual Debt Service ($4,250,126.44) = | $ 5,312,658.05 |
| Maximum Annual Debt Service = | $ 4,440,870.00 |
| Debt Service Reserve Fund (lesser of above) = | $ 4,440,870.00 |

- Cost of Issuance

# Bond Summary Statistics

| | |
|---|---|
| Dated Date | 5/6/2004 |
| Delivery Date | 5/6/2004 |
| Last Maturity | 5/1/2024 |
| | |
| Arbitrage Yield | 5.395% |
| True Interest Cost (TIC) | 5.452% |
| Net Interest Cost (NIC) | 5.497% |
| All-In TIC | 5.497% |
| Average Coupon | 5.458% |
| | |
| Average Life (years) | 12.80371111 |
| Duration of Issue (years) | 9.115007522 |

- Yield = Rate at which Present Value of debt service equals the Target Value on the Target Date

| | TIC | All-In TIC | Arbitrage Yield |
|---|---|---|---|
| **Par Value** | $ 50,000,000 | $ 50,000,000 | $ 50,000,000 |
| **+ Accrued Interest** | - | - | - |
| **+ Premium (Discount)** | - | - | - |
| **- Underwriter's Discount** | (250,000) | (250,000) | |
| **- Cost of Issuance Expense** | | (200,000) | |
| **- Other Amounts** | - | - | - |
| **Target Value** | $ 49,750,000 | $ 49,550,000 | $ 50,000,000 |
| **Target Date** | 5/6/2004 | 5/6/2004 | 5/6/2004 |
| **Yield** | 5.452% | 5.497% | 5.395% |

# Bond Summary Statistics

| | |
|---|---|
| Par Amount | $ 50,000,000.00 |
| Bond Proceeds | $ 50,000,000.00 |
| Total Interest | $ 34,943,499.20 |
| Net Interest | $ 35,193,499.20 |
| Total Debt Service | $ 84,943,499.20 |
| Maximum Annual Debt Service | $ 4,440,870.00 |
| Average Annual Debt Service | $ 4,250,126.44 |
| | |
| Underwriter's Fees (per $1000) | |
| Management Fee | $ - |
| Average Takedown | $ 4.00 |
| Other Fee | $ 1.00 |
| | --- |
| Total Underwriter's Discount | $ 5.00 |
| | |
| Bid Price | $ 99.50 |

| Bond Component | Par Value | Price | Average Coupon | Average Life |
|---|---|---|---|---|
| Serial Maturities | $ 16,940,000.00 | 100 | 4.098% | 6.673 |
| Term Bond Maturing in 2024 | $ 33,060,000.00 | 100 | 5.750% | 15.945 |
| | $ 50,000,000.00 | | | 12.803 |

# Bond Pricing

| | Maturity Date | Amount | Rate | Yield | Price |
|---|---|---|---|---|---|
| Serial Maturities | 5/1/2007 | $ 1,895,000.00 | 2.050% | 2.050% | 100 |
| | 5/1/2008 | $ 1,935,000.00 | 2.750% | 2.750% | 100 |
| | 5/1/2009 | $ 1,985,000.00 | 3.250% | 3.250% | 100 |
| | 5/1/2010 | $ 2,050,000.00 | 3.750% | 3.750% | 100 |
| | 5/1/2011 | $ 2,125,000.00 | 4.250% | 4.250% | 100 |
| | 5/1/2012 | $ 2,215,000.00 | 4.450% | 4.450% | 100 |
| | 5/1/2013 | $ 2,315,000.00 | 4.600% | 4.600% | 100 |
| | 5/1/2014 | $ 2,420,000.00 | 4.750% | 4.750% | 100 |
| Term Bond Maturing in 2024 | 5/1/2024 | $ 33,060,000.00 | 5.750% | 5.750% | 100 |

# Bond Debt Service

- After capitalized interest period, level debt service over 20 years

| Period Ending | Principal | Coupon | Interest | Debt Service | Annual Debt Service |
|---|---|---|---|---|---|
| 5/6/2004 | $ - | | $ - | $ - | $ - |
| 11/1/2004 | $ - | | $ 1,237,015.45 | $ 1,237,015.45 | $ - |
| 5/1/2005 | $ - | | $ 1,272,358.75 | $ 1,272,358.75 | $ 2,509,374.20 |
| 11/1/2005 | $ - | | $ 1,272,358.75 | $ 1,272,358.75 | $ - |
| 5/1/2006 | $ - | | $ 1,272,358.75 | $ 1,272,358.75 | $ 2,544,717.50 |
| 11/1/2006 | $ - | | $ 1,272,358.75 | $ 1,272,358.75 | $ - |
| 5/1/2007 | $ 1,895,000.00 | 2.050% | $ 1,272,358.75 | $ 3,167,358.75 | $ 4,439,717.50 |
| 11/1/2007 | $ - | | $ 1,252,935.00 | $ 1,252,935.00 | $ - |
| 5/1/2008 | $ 1,935,000.00 | 2.750% | $ 1,252,935.00 | $ 3,187,935.00 | $ 4,440,870.00 |
| 11/1/2008 | $ - | | $ 1,226,328.75 | $ 1,226,328.75 | $ - |
| 5/1/2009 | $ 1,985,000.00 | 3.250% | $ 1,226,328.75 | $ 3,211,328.75 | $ 4,437,657.50 |
| 11/1/2009 | $ - | | $ 1,194,072.50 | $ 1,194,072.50 | $ - |
| 5/1/2010 | $ 2,050,000.00 | 3.750% | $ 1,194,072.50 | $ 3,244,072.50 | $ 4,438,145.00 |
| . | . | . | . | . | . |
| . | . | . | . | . | . |
| . | . | . | . | . | . |
| 11/1/2023 | $ - | | $ 120,606.25 | $ 120,606.25 | $ - |
| 5/1/2024 | $ 4,195,000.00 | 5.750% | $ 120,606.25 | $ 4,315,606.25 | $ 4,436,212.50 |
| | $ 50,000,000.00 | | $ 34,943,499.20 | $ 84,943,499.20 | $ 84,943,499.20 |

# Net Debt Service

- Gross debt service less capitalized interest earnings on bond funds

| Period Ending | Total Debt Service | Debt Service Reserve Fund | Capitalized Interest Fund | Net Debt Service |
|---|---|---|---|---|
| 6/30/2005 | $ 2,509,374.20 | $ - | $ 2,509,374.20 | $ - |
| 6/30/2006 | $ 2,544,717.50 | $ 119,802.64 | $ 1,272,358.75 | $ 1,152,556.11 |
| 6/30/2007 | $ 4,439,717.50 | $ 239,605.28 | $ - | $ 4,200,112.22 |
| 6/30/2008 | $ 4,440,870.00 | $ 239,605.28 | $ - | $ 4,201,264.72 |
| 6/30/2009 | $ 4,437,657.50 | $ 239,605.28 | $ - | $ 4,198,052.22 |
| 6/30/2010 | $ 4,438,145.00 | $ 239,605.28 | $ - | $ 4,198,539.72 |
| . | . | . | . | . |
| . | . | . | . | . |
| . | . | . | . | . |
| 6/30/2023 | $ 4,439,487.50 | $ 239,605.28 | $ - | $ 4,199,882.22 |
| 6/30/2024 | $ 4,436,212.50 | $ 4,680,475.28 | $ - | $ (244,262.78) |
| | $ 84,943,499.20 | $ 8,873,567.68 | $ 3,781,732.95 | $ 72,288,198.57 |

# Project Fund

- 12 month construction period with equal monthly draws; net funded

| Date | Deposit | Interest @ 2.5% | Principal | Scheduled Draws | Balance |
|---|---|---|---|---|---|
| 5/6/2004 | $ 41,766,602.69 | $ - | $ 3,519,208.73 | $ 3,519,208.73 | $ 38,247,393.96 |
| 6/1/2004 | $ - | $ 66,047.10 | $ 3,453,161.63 | $ 3,519,208.73 | $ 34,794,232.33 |
| 7/1/2004 | $ - | $ 72,113.30 | $ 3,447,095.43 | $ 3,519,208.73 | $ 31,347,136.90 |
| 8/1/2004 | $ - | $ 64,968.97 | $ 3,454,239.76 | $ 3,519,208.73 | $ 27,892,897.14 |
| 9/1/2004 | $ - | $ 57,809.84 | $ 3,461,398.89 | $ 3,519,208.73 | $ 24,431,498.25 |
| 10/1/2004 | $ - | $ 50,635.86 | $ 3,468,572.87 | $ 3,519,208.73 | $ 20,962,925.38 |
| 11/1/2004 | $ - | $ 43,447.02 | $ 3,475,761.71 | $ 3,519,208.73 | $ 17,487,163.67 |
| 12/1/2004 | $ - | $ 36,243.28 | $ 3,482,965.45 | $ 3,519,208.73 | $ 14,004,198.22 |
| 1/1/2005 | $ - | $ 29,024.61 | $ 3,490,184.12 | $ 3,519,208.73 | $ 10,514,014.10 |
| 2/1/2005 | $ - | $ 21,790.98 | $ 3,497,417.75 | $ 3,519,208.73 | $ 7,016,596.35 |
| 3/1/2005 | $ - | $ 14,542.35 | $ 3,504,666.38 | $ 3,519,208.73 | $ 3,511,929.97 |
| 4/1/2005 | $ - | $ 7,278.70 | $ 3,511,929.97 | $ 3,519,208.67 | $ - |
| | $ 41,766,602.69 | $ 463,902.01 | $ 41,766,602.69 | $ 42,230,504.70 | |

# Reserve Funds – Capitalized Interest Fund

- 18 months: 12 month construction period plus six months cushion; net funded with earnings from capitalized interest fund and DSRF

| Date | Deposit | Interest @ 2.5% | Earnings from Debt Service Reserve Fund | Principal | Scheduled Draws | Balance |
|---|---|---|---|---|---|---|
| 5/6/2004 | $ 3,342,527.31 | $ - | $ - | $ - | $ - | $ 3,342,527.31 |
| 11/1/2004 | $ - | $ 40,613.97 | $ 116,474.79 | $ 1,079,926.69 | $ 1,237,015.45 | $ 2,262,600.62 |
| 5/1/2005 | $ - | $ 28,282.51 | $ 119,802.64 | $ 1,124,273.60 | $ 1,272,358.75 | $ 1,138,327.02 |
| 11/1/2005 | $ - | $ 14,229.09 | $ 119,802.64 | $ 1,138,327.02 | $ 1,272,358.75 | $ - |
| | $ 3,342,527.31 | $ 83,125.57 | $ 356,080.07 | $ 3,342,527.31 | $ 3,781,732.95 | |

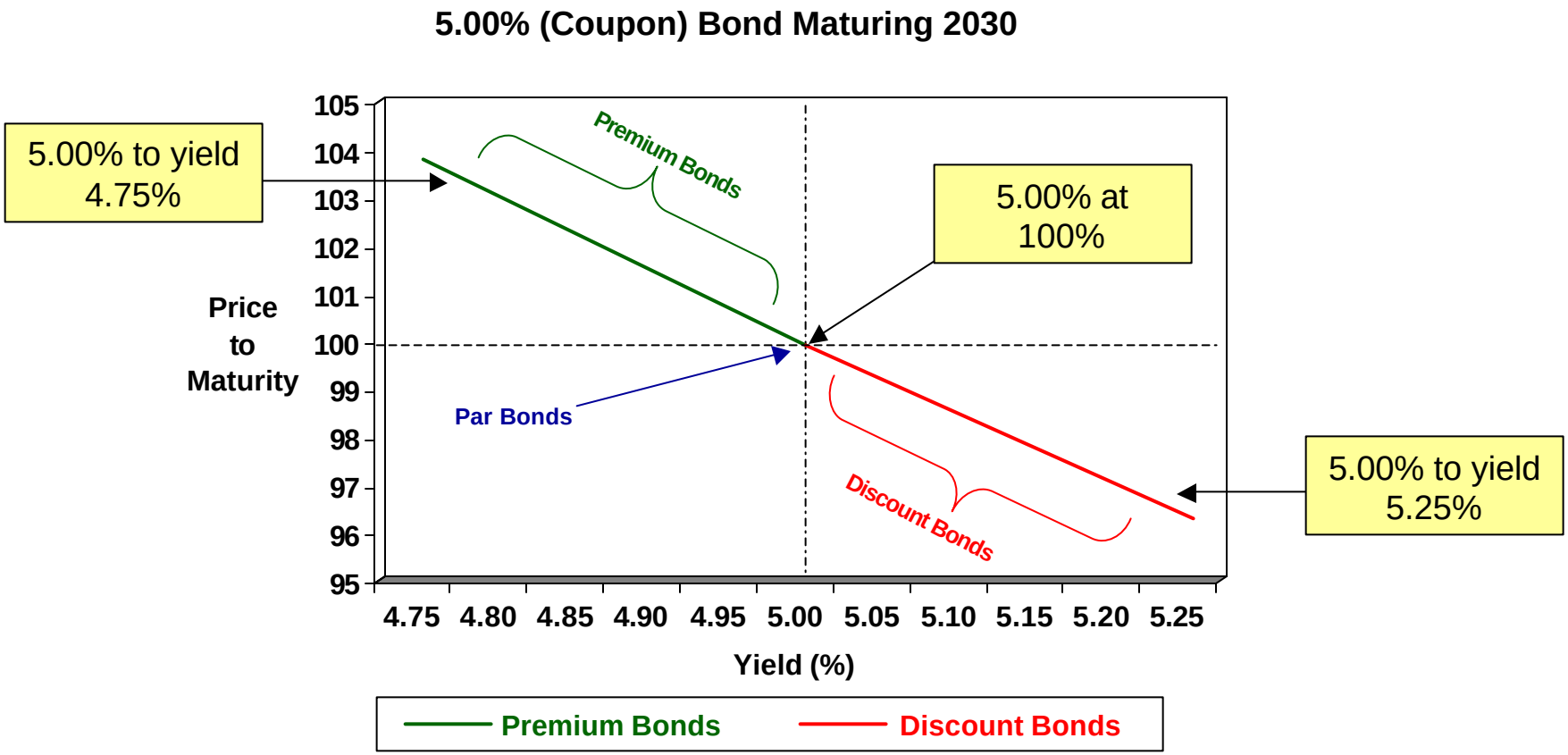
# Debt Service Reserve Fund

- Gross funded with earnings at the arbitrage yield; earnings flow to capitalized interest fund during capitalized interest period

| Date | Deposit | Interest @ 5.395458% | Principal | Capitalized Interest Fund | Debt Service | Balance |
|---|---|---|---|---|---|---|
| 5/6/2004 | $ 4,440,870.00 | $ - | $ - | $ - | $ - | $ 4,440,870.00 |
| 11/1/2004 | $ - | $ 116,474.79 | $ - | $ (116,474.79) | $ - | $ 4,440,870.00 |
| 5/1/2005 | $ - | $ 119,802.64 | $ - | $ (119,802.64) | $ - | $ 4,440,870.00 |
| 11/1/2005 | $ - | $ 119,802.64 | $ - | $ (119,802.64) | $ - | $ 4,440,870.00 |
| 5/1/2006 | $ - | $ 119,802.64 | $ - | $ - | $ (119,802.64) | $ 4,440,870.00 |
| 11/1/2006 | $ - | $ 119,802.64 | $ - | $ - | $ (119,802.64) | $ 4,440,870.00 |
| 5/1/2007 | $ - | $ 119,802.64 | $ - | $ - | $ (119,802.64) | $ 4,440,870.00 |
| 11/1/2007 | $ - | $ 119,802.64 | $ - | $ - | $ (119,802.64) | $ 4,440,870.00 |
| 5/1/2008 | $ - | $ 119,802.64 | $ - | $ - | $ (119,802.64) | $ 4,440,870.00 |
| 11/1/2008 | $ - | $ 119,802.64 | $ - | $ - | $ (119,802.64) | $ 4,440,870.00 |
| 5/1/2009 | $ - | $ 119,802.64 | $ - | $ - | $ (119,802.64) | $ 4,440,870.00 |
| 11/1/2009 | $ - | $ 119,802.64 | $ - | $ - | $ (119,802.64) | $ 4,440,870.00 |
| 5/1/2010 | $ - | $ 119,802.64 | $ - | $ - | $ (119,802.64) | $ 4,440,870.00 |
| 11/1/2010 | $ - | $ 119,802.64 | $ - | $ - | $ (119,802.64) | $ 4,440,870.00 |
| . | . | . | . | . | . | . |
| . | . | . | . | . | . | . |
| . | . | . | . | . | . | . |
| 11/1/2023 | $ - | $ 119,802.64 | $ - | $ - | $ (119,802.64) | $ 4,440,870.00 |
| 5/1/2024 | $ - | $ 119,802.64 | $ 4,440,870.00 | $ - | $ (4,560,672.64) | $ - |
| | $ 4,440,870.00 | $ 4,788,777.75 | $ 4,440,870.00 | $ (356,080.07) | $ (8,873,567.68) | |

# Prices Versus Yields

- Prices and Yields are inversely correlated:
  - Yields ↑ Prices ↓
  - Yields ↓ Prices ↑



# Prices Versus Yields

- Example: 5.00% Bond Maturing on May 1, 2030

| Coupon | Yield | Price | Bond Issued for $100 of Proceeds |
|---|---|---|---|
| 5.00% | 4.75% | 103.710 | 96.423 |
| 5.00% | 4.80% | 102.952 | 97.133 |
| 5.00% | 4.85% | 102.203 | 97.844 |
| 5.00% | 4.90% | 101.461 | 98.560 |
| 5.00% | 4.95% | 100.726 | 99.279 |
| 5.00% | 5.00% | 100.000 | 100.000 |
| 5.00% | 5.05% | 99.280 | 100.725 |
| 5.00% | 5.10% | 98.568 | 101.453 |
| 5.00% | 5.15% | 97.863 | 102.184 |
| 5.00% | 5.20% | 97.166 | 102.917 |
| 5.00% | 5.25% | 96.475 | 103.654 |

## Callable vs. Non-Callable Bonds

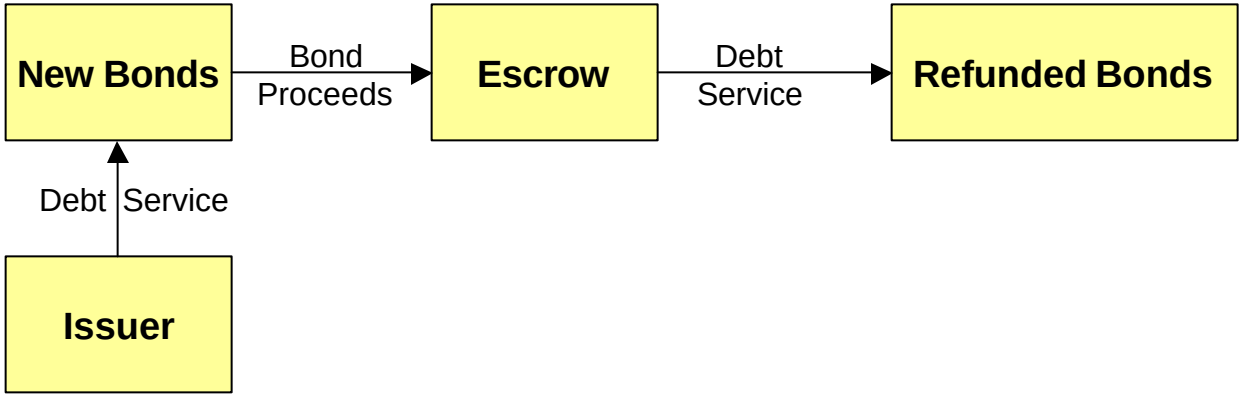
- Bonds may be issued with the ability to “pre-pay”, “redeem” or “call” the bonds prior to their scheduled maturity
- Redemptions are typically optional, mandatory or extraordinary
  - Optional redemption allows an issuer to defease or refund the bonds prior to scheduled maturity
    - Under current market conditions, the standard optional redemption provision is 10 years from issuance at par (100)
    - Historically, the call premium has been as high as 102, declining to a par call in two years
  - Typically, extraordinary redemption allows for prepayment in the event of damage or destruction
  - Mandatory redemption allows for annual sinking fund payments on term bonds
- Note: The MSRB pricing formula requires special pricing for callable premium bonds given the likelihood of early redemption
  - The price is calculated to each call date and to maturity
  - Whichever calculation produces the lowest price is the price to investors

# Example: Refunding Issue for the Pasadena Historical Museum Project

- It is five years later and interest rates have declined by approximately 150 bps
- The City has hired us to refund their bonds to generate savings
- The bonds are callable on May 1, 2014 at par
- Investors are demanding premium and discount bonds

# Refunding Bond Mechanics

- Refunding bonds are issued to fund an escrow to:
  - Pay scheduled principal and interest prior to the call date of the Refunded Bonds
  - Pay remaining principal plus redemption premium, if any, on the Refunded Bonds call date of the Refunded Bonds
- Issuer pays debt service on the Refunding bonds and the escrow pays debt service on Refunded Bonds




- Until the call date of the Refunded Bonds, both series of bonds remain outstanding

# Refunding Bond Mechanics

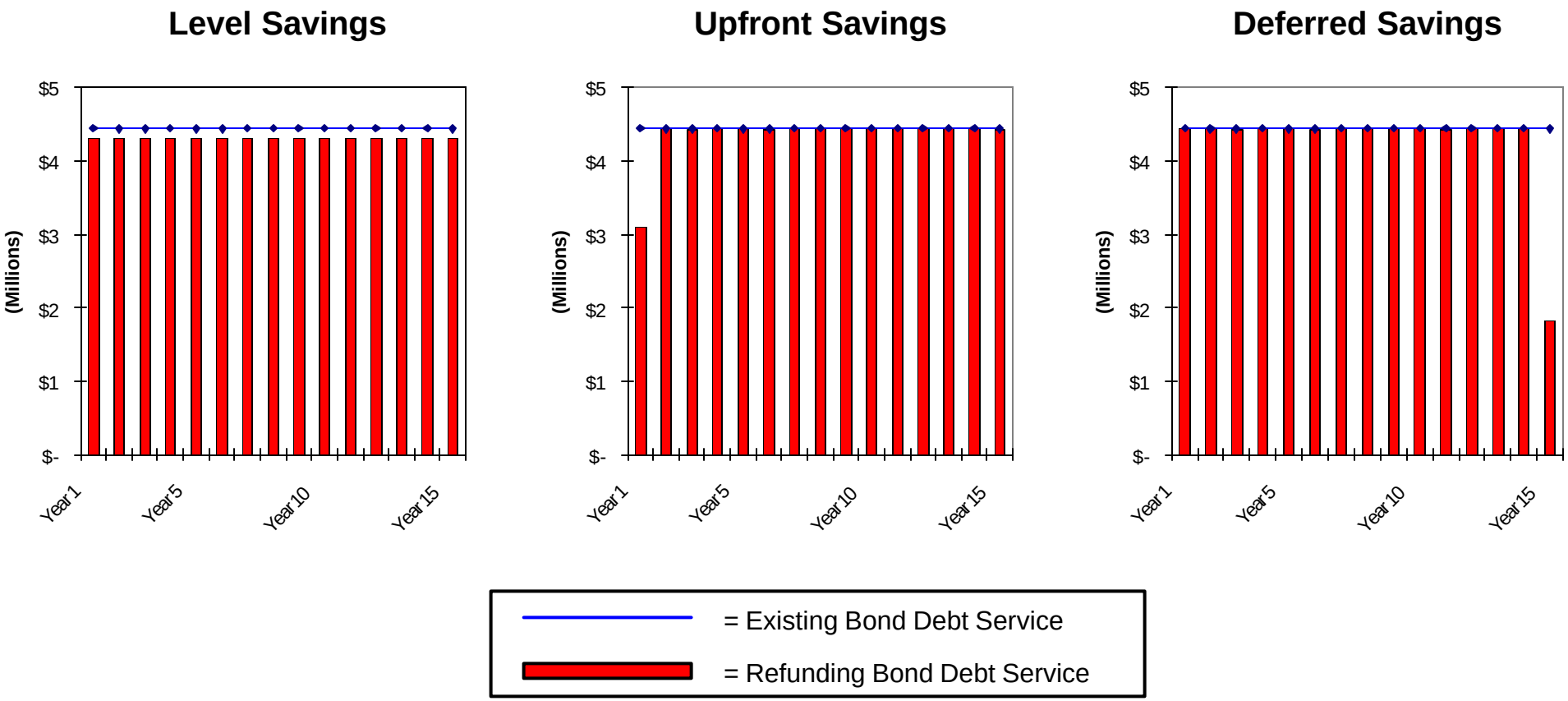
- Refunding savings is generated by the difference in interest rates on the new refunding bonds and the old refunded bonds
- Savings are also affected by how much it costs to pay off the old bonds (i.e. to fund the escrow account)
  - Under tax law, the interest rate the securities in the escrow can earn is limited to the arbitrage yield of the refunding bonds
  - Depending on market conditions, escrow investments may not be able to earn the arbitrage yield
    - Any negative difference between earnings at the arbitrage yield and actual earnings on the escrow is referred to as negative arbitrage
    - There may be a opportunities to “make up” negative arbitrage from earnings on other bond funds, such as on reserve funds, or in any new money bond funds issued along with the refunding bonds

# Refunding Bond Mechanics

- Savings can be structured in various patterns, i.e., level, upfront or deferred
  - An upfront savings structure provides near term savings with remaining debt service equal to prior debt service
  - A level savings structure provides equal annual savings through maturity.
  - A deferred savings structure maintains prior debt service in the near term and shortens the final maturity

# Refunding Bond Mechanics

- The pattern of savings depends upon legal structure and/or issuer preferences



# Refunding Bond Issue Financing Reports

- In addition to the financing reports for a new financing, there are reports specific to a refunding issue:
  - Summary of Refunding Results
  - Savings
  - Summary of Refunded Bonds
  - Escrow Reports
    - Escrow Requirements
    - Escrow Descriptions
    - Escrow Sufficiency
    - Escrow Statistics

# Sources and Uses

| SOURCES | |
|---|---|
| Bond Proceeds | |
| Par Amount | $45,435,000.00 |
| Net Premium | $ 2,091,217.00 |
| Other Sources of Funds | |
| Existing DSRF | $ 4,440,870.00 |
| Total | $ 51,967,087.00 |

# Sources and Uses

| USES | | |
|---|---|---|
| Refunding Escrow Deposits | | |
| Cash Deposit | $ | 662.50 |
| Open Market Purchases | $ | 47,231,930.60 |
| Other Fund Deposits | | |
| Debt Service Reserve Fund | $ | 4,305,250.00 |
| Delivery Date Expenses | | |
| Cost of Issuance | $ | 200,000.00 |
| Underwriter's Discount | $ | 227,175.00 |
| Additional Proceeds | $ | 2,068.90 |
| Total | $ | 51,967,087.00 |

# Bond Pricing

| | Maturity Date | Amount | Rate | Yield | Price | |
|---|---|---|---|---|---|---|
| Serial Maturities | 5/1/2010 | $ 2,360,000.00 | 2.000% | 1.150% | 100.831 | |
| | 5/1/2011 | $ 2,380,000.00 | 2.000% | 1.600% | 100.778 | |
| | 5/1/2012 | $ 2,430,000.00 | 2.000% | 2.050% | 99.855 | |
| | 5/1/2013 | $ 2,480,000.00 | 3.250% | 2.750% | 101.875 | |
| | 5/1/2014 | $ 2,560,000.00 | 3.250% | 3.250% | 100.000 | |
| Term Bond Maturing in 2024 | 5/1/2024 | $ 33,225,000.00 | 5.000% | 4.250% | 106.050 | C |

C= Priced to Call Date of 5/1/2019 at Par

# Summary of Refunding Results

| | | |
|---|---|---|
| Dated Date | | 5/6/2009 |
| Delivery Date | | 5/6/2009 |
| Arbitrage yield | | 4.000% |
| Escrow yield | | 3.862% |
| | | |
| Bond Par Amount | $ | 45,435,000.00 |
| True Interest Cost | | 4.176% |
| Net Interest Cost | | 4.325% |
| Average Coupon | | 4.793% |
| Average Life | | 8.775150398 |
| | | |
| Par amount of refunded bonds | $ | 44,185,000.00 |
| Average coupon of refunded bonds | | 5.645% |
| Average life of refunded bonds | | 8.962121069 |
| | | |
| PV of prior debt to 05/06/2009 @ 4.000430% | $ | 49,408,671.41 |
| Net PV Savings | $ | 1,391,307.70 |
| Percentage savings of refunded bonds | | 3.149% |
| Percentage savings of refunding bonds | | 3.062% |

# Savings

- Savings are structured to be level on an annual basis

| Fiscal Year | Prior Debt Service | Refunding Debt Service | Annual Savings | Present Value to 05/06/2009 @ 4.0004303% |
|---|---|---|---|---|
| 2010 | $ 4,438,145.00 | $ 4,301,110.42 | $ 137,034.58 | $ 136,347.80 |
| 2011 | $ 4,436,270.00 | $ 4,301,250.00 | $ 135,020.00 | $ 128,410.72 |
| 2012 | $ 4,435,957.50 | $ 4,303,650.00 | $ 132,307.50 | $ 120,634.42 |
| 2013 | $ 4,437,390.00 | $ 4,305,050.00 | $ 132,340.00 | $ 115,550.56 |
| 2014 | $ 4,435,900.00 | $ 4,304,450.00 | $ 131,450.00 | $ 110,120.10 |
| 2015 | $ 4,435,950.00 | $ 4,301,250.00 | $ 134,700.00 | $ 108,157.00 |
| 2016 | $ 4,440,187.50 | $ 4,304,250.00 | $ 135,937.50 | $ 104,790.68 |
| 2017 | $ 4,440,800.00 | $ 4,300,500.00 | $ 140,300.00 | $ 103,786.61 |
| 2018 | $ 4,437,500.00 | $ 4,305,000.00 | $ 132,500.00 | $ 94,167.29 |
| 2019 | $ 4,440,000.00 | $ 4,302,000.00 | $ 138,000.00 | $ 94,082.19 |
| 2020 | $ 4,437,437.50 | $ 4,301,500.00 | $ 135,937.50 | $ 88,950.90 |
| 2021 | $ 4,439,525.00 | $ 4,303,000.00 | $ 136,525.00 | $ 85,710.05 |
| 2022 | $ 4,440,400.00 | $ 4,301,000.00 | $ 139,400.00 | $ 83,938.17 |
| 2023 | $ 4,439,487.50 | $ 4,305,250.00 | $ 134,237.50 | $ 77,538.40 |
| 2024 | $ 4,436,212.50 | $ 4,305,000.00 | $ 131,212.50 | $ 72,673.91 |
| | $ 66,571,162.50 | $ 64,544,260.42 | $ 2,026,902.08 | $ 1,524,858.80 |

**Savings Summary**

| | |
|---|---|
| PV of savings from cash flow | $1,524,858.80 |
| Less: Prior funds on hand | $(4,440,870.00) |
| Plus: Refunding funds on hand | $4,307,318.90 |
| | --- |
| Net PV Savings | $1,391,307.70 |

# Summary of Refunded Bonds

| Maturity Date | Interest Rate | Par Amount | Call Date | Call Price |
|---|---|---|---|---|
| 5/1/2010 | 3.750% | $ 2,050,000.00 | | |
| 5/1/2011 | 4.250% | $ 2,125,000.00 | | |
| 5/1/2012 | 4.450% | $ 2,215,000.00 | | |
| 5/1/2013 | 4.600% | $ 2,315,000.00 | | |
| 5/1/2014 | 4.750% | $ 2,420,000.00 | | |
| 5/1/2024 | 5.750% | $ 33,060,000.00 | 5/1/2014 | 100.000 |
| | | $ 44,185,000.00 | | |

# Escrow Requirements

| Period Ending | Principal | Interest | Principal Redeemed | Total |
|---|---|---|---|---|
| 11/1/2009 | $ - | $ 1,194,072.50 | $ - | $ 1,194,072.50 |
| 5/1/2010 | $ 2,050,000.00 | $ 1,194,072.50 | $ - | $ 3,244,072.50 |
| 11/1/2010 | $ - | $ 1,155,635.00 | $ - | $ 1,155,635.00 |
| 5/1/2011 | $ 2,125,000.00 | $ 1,155,635.00 | $ - | $ 3,280,635.00 |
| 11/1/2011 | $ - | $ 1,110,478.75 | $ - | $ 1,110,478.75 |
| 5/1/2012 | $ 2,215,000.00 | $ 1,110,478.75 | $ - | $ 3,325,478.75 |
| 11/1/2012 | $ - | $ 1,061,195.00 | $ - | $ 1,061,195.00 |
| 5/1/2013 | $ 2,315,000.00 | $ 1,061,195.00 | $ - | $ 3,376,195.00 |
| 11/1/2013 | $ - | $ 1,007,950.00 | $ - | $ 1,007,950.00 |
| 5/1/2014 | $ 2,420,000.00 | $ 1,007,950.00 | $ 33,060,000.00 | $ 36,487,950.00 |
| | $ 11,125,000.00 | $ 11,058,662.50 | $ 33,060,000.00 | $ 55,243,662.50 |

# Escrow Descriptions

| Purchase Date | Type of Security | Type of SLGS | Maturity Date | Par Amount | Yield | Price | Interest Class | Interest Frequency | Interest Day Basis |
|---|---|---|---|---|---|---|---|---|---|
| 5/6/2009 | TSTRIP-I | | 8/15/2009 | $ 1,194,000 | 2.997% | 99.171 | Zero Coupon | Semiannual | ACT/ACT |
| 5/6/2009 | TSTRIP-I | | 2/15/2010 | $ 3,244,000 | 3.224% | 97.539 | Zero Coupon | Semiannual | ACT/ACT |
| 5/6/2009 | TSTRIP-I | | 8/15/2010 | $ 1,156,000 | 3.344% | 95.847 | Zero Coupon | Semiannual | ACT/ACT |
| 5/6/2009 | TSTRIP-I | | 2/15/2011 | $ 3,280,000 | 3.497% | 94.019 | Zero Coupon | Semiannual | ACT/ACT |
| 5/6/2009 | TSTRIP-I | | 8/15/2011 | $ 1,111,000 | 3.619% | 92.151 | Zero Coupon | Semiannual | ACT/ACT |
| 5/6/2009 | TSTRIP-I | | 2/15/2012 | $ 3,325,000 | 3.762% | 90.161 | Zero Coupon | Semiannual | ACT/ACT |
| 5/6/2009 | TSTRIP-I | | 8/15/2012 | $ 1,061,000 | 3.882% | 88.155 | Zero Coupon | Semiannual | ACT/ACT |
| 5/6/2009 | TSTRIP-I | | 2/15/2013 | $ 3,377,000 | 3.988% | 86.138 | Zero Coupon | Semiannual | ACT/ACT |
| 5/6/2009 | TSTRIP-I | | 8/15/2013 | $ 1,008,000 | 4.083% | 84.118 | Zero Coupon | Semiannual | ACT/ACT |
| 5/6/2009 | TSTRIP-I | | 2/15/2014 | $ 36,487,000 | 4.154% | 82.161 | Zero Coupon | Semiannual | ACT/ACT |
| 8/15/2009 | SLG | Rollover Cert | 11/1/2009 | $ 1,194,662 | | | | | |
| 2/15/2010 | SLG | Rollover Cert | 5/1/2010 | $ 3,244,590 | | | | | |
| 8/15/2010 | SLG | Rollover Cert | 11/1/2010 | $ 1,156,517 | | | | | |
| 2/15/2011 | SLG | Rollover Cert | 5/1/2011 | $ 3,280,882 | | | | | |
| 8/15/2011 | SLG | Rollover Cert | 11/1/2011 | $ 1,111,247 | | | | | |
| 2/15/2012 | SLG | Rollover Cert | 5/1/2012 | $ 3,325,768 | | | | | |
| 8/15/2012 | SLG | Rollover Cert | 11/1/2012 | $ 1,061,290 | | | | | |
| 2/15/2013 | SLG | Rollover Cert | 5/1/2013 | $ 3,377,095 | | | | | |
| 8/15/2013 | SLG | Rollover Cert | 11/1/2013 | $ 1,008,900 | | | | | |
| 2/15/2014 | SLG | Rollover Cert | 5/1/2014 | $ 36,487,950 | | | | | |

# Escrow Sufficiency

| Date | Escrow Requirement | Net Escrow Receipts | Excess Receipts | Excess Balance |
|---|---|---|---|---|
| 5/6/2009 | $ - | $ 662.50 | $ 662.50 | $ 662.50 |
| 8/15/2009 | $ - | $ (662.00) | $ (662.00) | $ 0.50 |
| 11/1/2009 | $ 1,194,072.50 | $ 1,194,662.00 | $ 589.50 | $ 590.00 |
| 2/15/2010 | $ - | $ (590.00) | $ (590.00) | $ - |
| 5/1/2010 | $ 3,244,072.50 | $ 3,244,590.00 | $ 517.50 | $ 517.50 |
| 8/15/2010 | $ - | $ (517.00) | $ (517.00) | $ 0.50 |
| 11/1/2010 | $ 1,155,635.00 | $ 1,156,517.00 | $ 882.00 | $ 882.50 |
| 2/15/2011 | $ - | $ (882.00) | $ (882.00) | $ 0.50 |
| 5/1/2011 | $ 3,280,635.00 | $ 3,280,882.00 | $ 247.00 | $ 247.50 |
| 8/15/2011 | $ - | $ (247.00) | $ (247.00) | $ 0.50 |
| 11/1/2011 | $ 1,110,478.75 | $ 1,111,247.00 | $ 768.25 | $ 768.75 |
| 2/15/2012 | $ - | $ (768.00) | $ (768.00) | $ 0.75 |
| 5/1/2012 | $ 3,325,478.75 | $ 3,325,768.00 | $ 289.25 | $ 290.00 |
| 8/15/2012 | $ - | $ (290.00) | $ (290.00) | $ - |
| 11/1/2012 | $ 1,061,195.00 | $ 1,061,290.00 | $ 95.00 | $ 95.00 |
| 2/15/2013 | $ - | $ (95.00) | $ (95.00) | $ - |
| 5/1/2013 | $ 3,376,195.00 | $ 3,377,095.00 | $ 900.00 | $ 900.00 |
| 8/15/2013 | $ - | $ (900.00) | $ (900.00) | $ - |
| 11/1/2013 | $ 1,007,950.00 | $ 1,008,900.00 | $ 950.00 | $ 950.00 |
| 2/15/2014 | $ - | $ (950.00) | $ (950.00) | $ - |
| 5/1/2014 | $ 36,487,950.00 | $ 36,487,950.00 | $ - | $ - |

# Escrow Statistics

| | Total Escrow Cost | Modified Duration (years) | Yield to Receipt Date | Yield to Disbursement Date | Perfect Escrow Cost | Value of Negative Arbitrage | Cost of Dead Time |
|---|---|---|---|---|---|---|---|
| Global Proceeds Escrow | $ 47,232,593.10 | 3.9801059 | 0.0386239 | 0.0386236 | $ 46,973,913.80 | $ 258,618.19 | 61.11 |

# The End

- Grand opening of the museum!