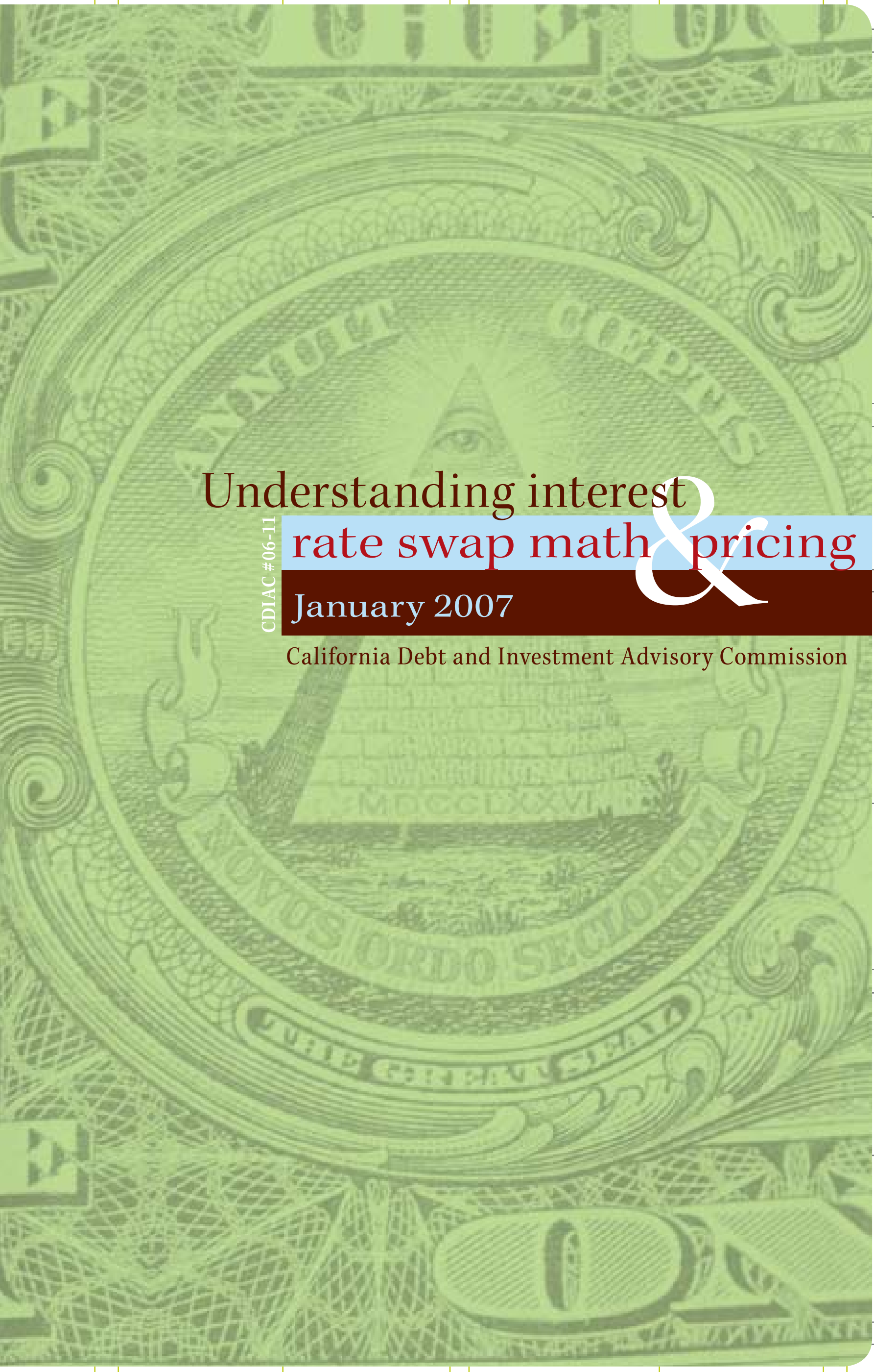

Understanding interest rate swap math & pricing

CDIAC #06-11

January 2007

California Debt and Investment Advisory Commission

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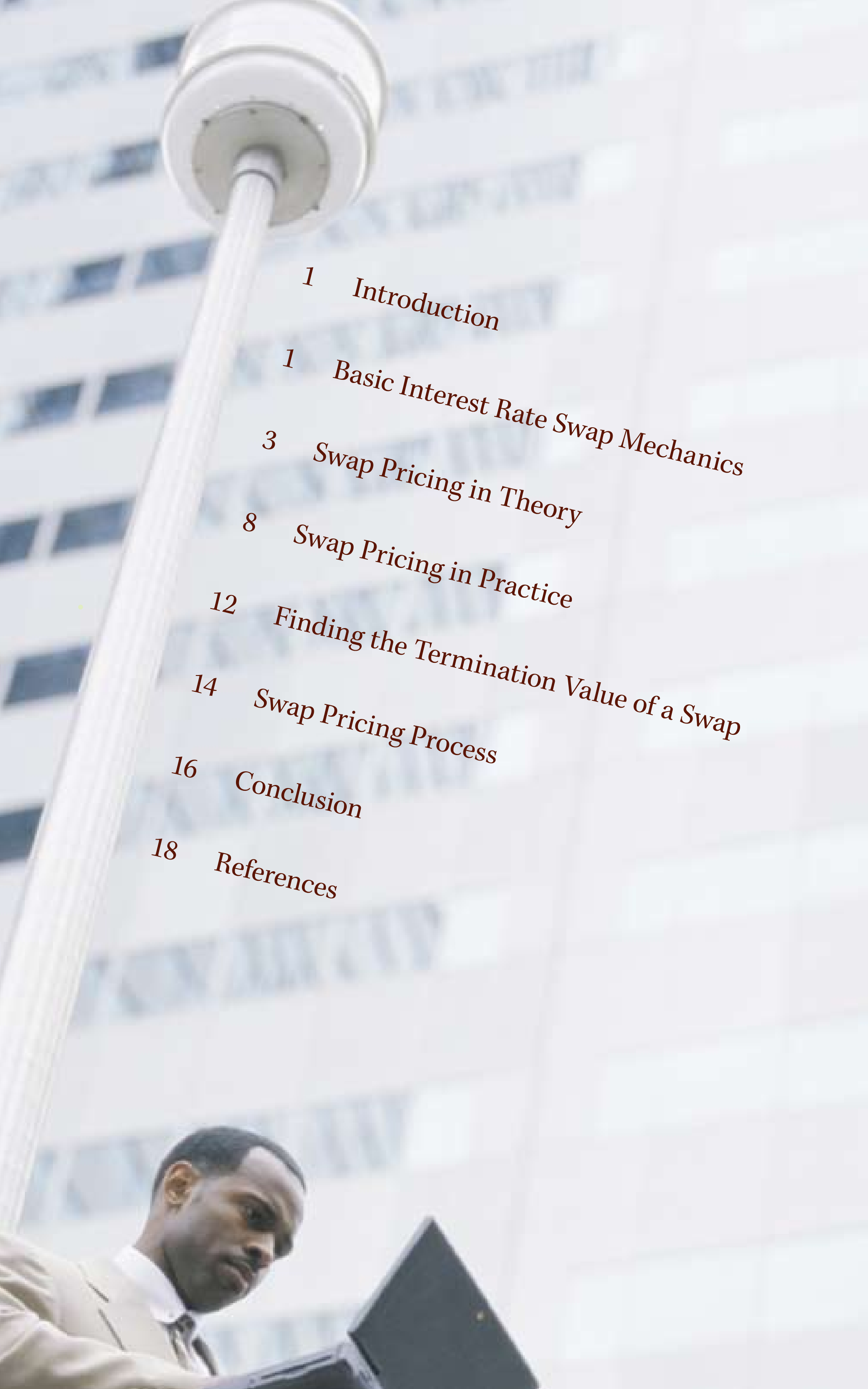
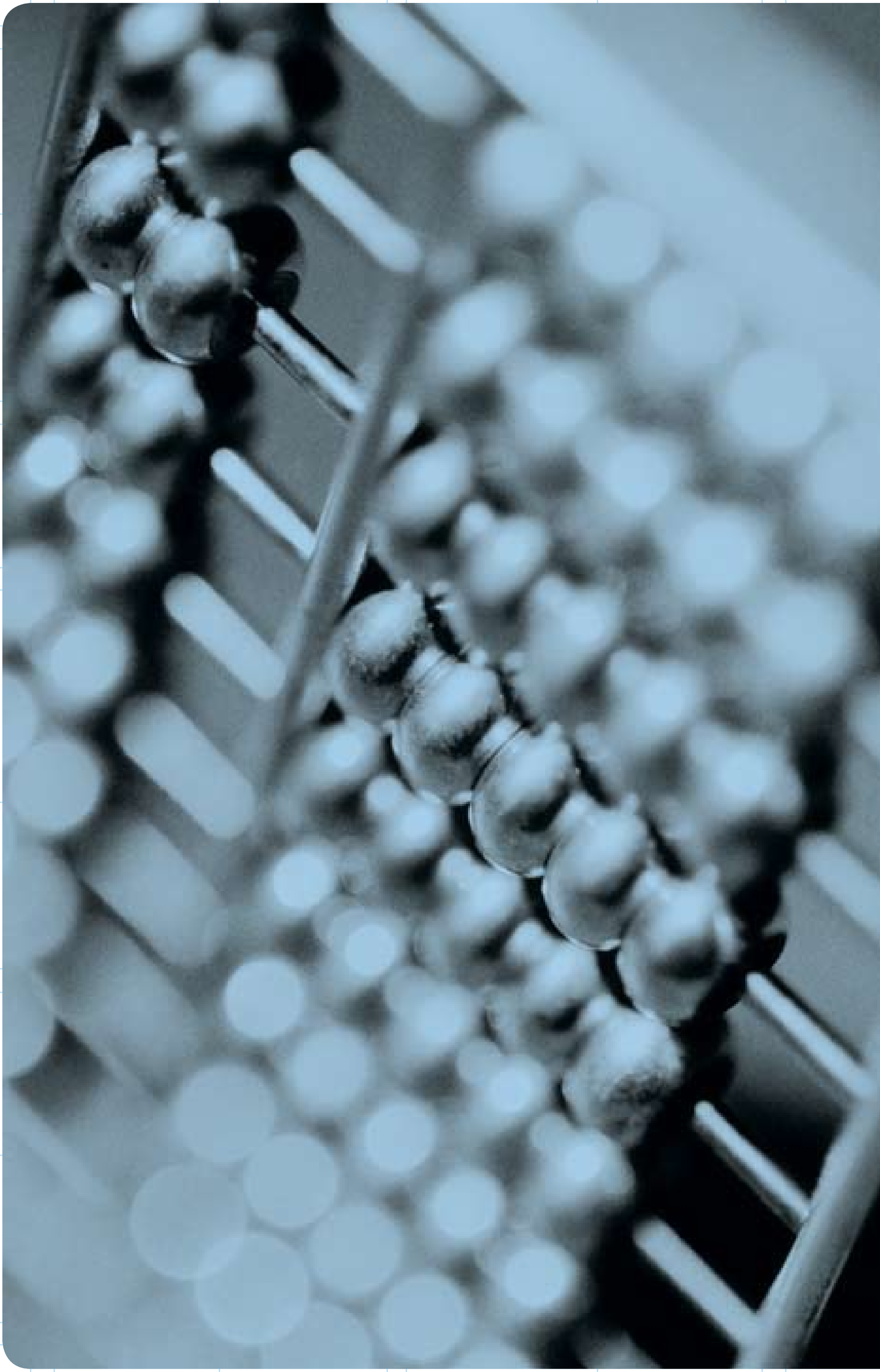




Introduction

As California local agencies are becoming involved in the interest rate swap market, knowledge of the basics of pricing swaps may assist issuers to better understand initial, mark-to-market, and termination costs associated with their swap programs.

This report is intended to provide treasury managers and staff with a basic overview of swap math and related pricing conventions. It provides information on the interest rate swap market, the swap dealer's pricing and sales conventions, the relevant indices needed to determine pricing, formulas for and examples of pricing, and a review of variables that have an affect on market and termination pricing of an existing swap.[1]

Basic Interest Rate Swap Mechanics

An interest rate swap is a contractual arrangement between two parties, often referred to as "counterparties". As shown in Figure 1, the counterparties (in this example, a financial institution and an issuer) agree to exchange payments based on a defined principal amount, for a fixed period of time.

In an interest rate swap, the principal amount is not actually exchanged between the counterparties, rather, interest payments are exchanged based on a "notional amount" or "notional principal." Interest rate swaps do not generate

[1] For those interested in a basic overview of interest rate swaps, the California Debt and Investment Advisory Commission (CDIAC) also has published *Fundamentals of Interest Rate Swaps* and *20 Questions for Municipal Interest Rate Swap Issuers*. These publications are available on the CDIAC website at www.treasurer.ca.gov/cdiac.

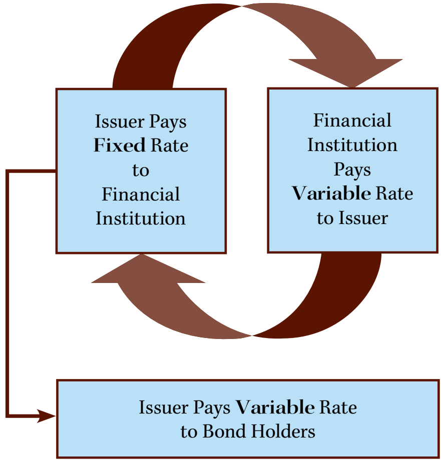


Figure 1

new sources of funding themselves; rather, they convert one interest rate basis to a different rate basis (e.g., from a floating or variable interest rate basis to a fixed interest rate basis, or vice versa). These "plain vanilla" swaps are by far the most common type of interest rate swaps.

Typically, payments made by one counterparty are based on a floating rate of interest, such as the London Inter Bank Offered Rate (LIBOR) or the Securities Industry and Financial Markets Association (SIFMA) Municipal Swap Index[2], while payments made by the other counterparty are based on a fixed rate of interest, normally expressed as a spread over U.S. Treasury bonds of a similar maturity.

The maturity, or "tenor," of a fixed-to-floating interest rate swap is usually between one and fifteen years. By convention, a fixed-rate payer is designated as the buyer of the swap, while the floating-rate payer is the seller of the swap.

Swaps vary widely with respect to underlying asset, maturity, style, and contingency provisions. Negotiated terms

[2] Formerly known as the Bond Market Association (BMA) Municipal Swap Index.

include starting and ending dates, settlement frequency, notional amount on which swap payments are based, and published reference rates on which swap payments are determined.

Swap Pricing in Theory

Interest rate swap terms typically are set so that the present value of the counterparty payments is at least equal to the present value of the payments to be received. Present value is a way of comparing the value of cash flows now with the value of cash flows in the future. A dollar today is worth more than a dollar in the future because cash flows available today can be invested and grown.

The basic premise to an interest rate swap is that the counterparty choosing to pay the fixed rate and the counterparty choosing to pay the floating rate each assume they will gain some advantage in doing so, depending on the swap rate. Their assumptions will be based on their needs and their estimates of the level and changes in interest rates during the period of the swap contract.

Because an interest rate swap is just a series of cash flows occurring at known future dates, it can be valued by simply summing the present value of each of these cash flows. In order to calculate the present value of each cash flow, it is necessary to first estimate the correct discount factor (df) for each period (t) on which a cash flow occurs. Discount factors are derived from investors' perceptions of interest rates in the future and are calculated using forward rates such as LIBOR. The following formula calculates a theoretical rate (known as the "Swap Rate") for the fixed component of the swap contract:

$$\text{Theoretical Swap Rate} = \frac{\text{Present value of the floating-rate payments}}{\sum_{t=1}^{N} \text{Notional principal} \times (\text{days}_t/360) \times \text{df}_t}$$

Consider the following example:

A municipal issuer and counterparty agree to a $100 million "plain vanilla" swap starting in January 2006 that calls for a 3-year maturity with the municipal issuer paying the Swap Rate (fixed rate) to the counterparty and the counterparty paying 6-month LIBOR (floating rate) to the issuer. Using the above formula, the Swap Rate can be calculated by using the 6-month LIBOR "futures" rate to estimate the present value of the floating component payments. Payments are assumed to be made on a semi-annual basis (i.e., 180-day periods). The above formula, shown as a step-by-step example, follows:

Step 1 – Calculate Numerator

The first step is to calculate the present value (PV) of the floating-rate payments.

This is done by forecasting each semi-annual payment using the LIBOR forward (futures) rates for the next three years. The following table illustrates the calculations based on actual semi-annual payments.[3]

[3] LIBOR forward rates are available through financial information services including Bloomberg, the *Wall Street Journal*, and the *Financial Times* of London.

Time Period	Period Number	Days in Period	Annual Forward Rate	Semi-annual Forward Period Rate	Actual Floating Rate Payment at End Period	Floating Rate Forward Discount Factor	PV of Floating Rate Payment at End of Period
(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)
1/06-6/06	1	180	4.00%	2.000%	$2,000,000	0.9804	$1,960,800
7/06-12/06	2	180	4.25%	2.125%	$2,125,000	0.9600	$2,040,000
1/07-6/07	3	180	4.50%	2.250%	$2,250,000	0.9389	$2,112,525
7/07-12/07	4	180	4.75%	2.375%	$2,375,000	0.9171	$2,178,113
1/08-6/08	5	180	5.00%	2.500%	$2,500,000	0.8947	$2,236,750
7/08-12/08	6	180	5.25%	2.625%	$2,625,000	0.8718	$2,288,475
PV of Floating Rate Payments=							**$12,816,663**

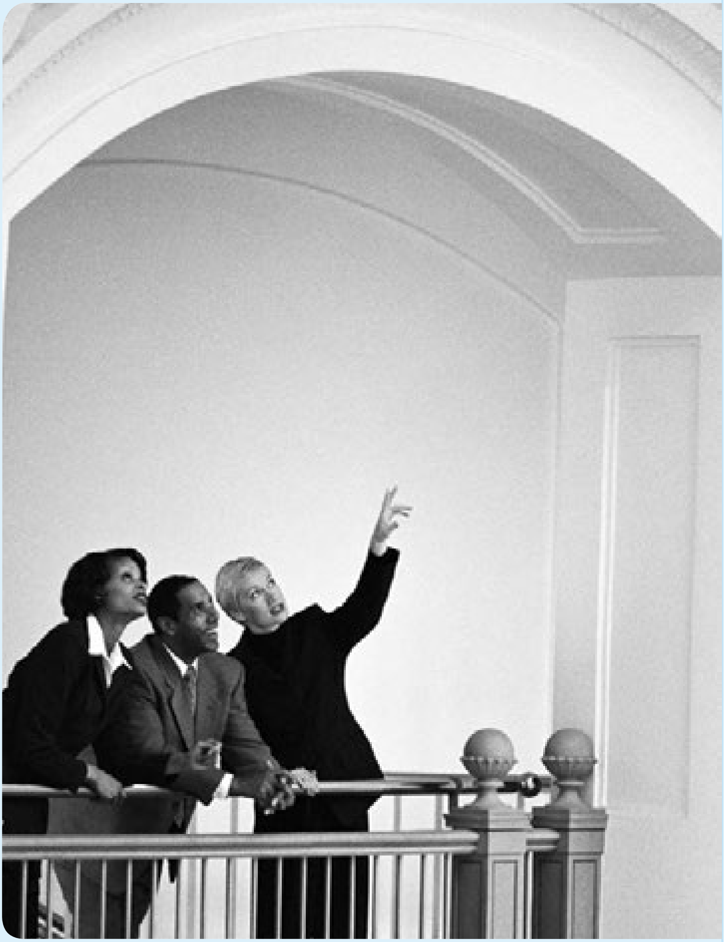
Column Description

A= Period the interest rate is in effect
B= Period number (t)
C= Number of days in the period (semi-annual=180 days)
D= Annual interest rate for the future period from financial publications
E= Semi-annual rate for the future period (D/2)
F= Actual forecasted payment (E × $100,000,000)
G= Discount factor=1/[(forward rate for period 1)(forward rate for period 2)…(forward rate for period t)]
H= PV of floating rate payments (F × G)

Step 2 – Calculate Denominator

As with the floating-rate payments, LIBOR forward rates are used to discount the notional principal for the three-year period. The PV of the notional principal is calculated by multiplying the notional principal by the days in the period and the floating-rate forward discount factor.

The following table illustrates the calculations for this example:



Time Period	Period Number	Days in Period	Annual Forward Rate	Semi-annual Forward Period Rate	Notional Principal	Floating Rate Forward Discount Factor	PV of Notional Principal
(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)
1/06-6/06	1	180	4.00%	2.000%	$100,000,000	0.9804	$49,020,000
7/06-12/06	2	180	4.25%	2.125%	$100,000,000	0.9600	$48,000,000
1/07-6/07	3	180	4.50%	2.250%	$100,000,000	0.9389	$46,945,000
7/07-12/07	4	180	4.75%	2.375%	$100,000,000	0.9171	$45,855,000
1/08-6/08	5	180	5.00%	2.500%	$100,000,000	0.8947	$44,735,000
7/08-12/08	6	180	5.25%	2.625%	$100,000,000	0.8718	$43,590,000
						PV of Notional Principal=	**$278,145,000**

Column Description

A= Period the interest rate is in effect
B= Period number (t)
C= Number of days in the period (semi-annual=180 days)
D= Annual interest rate for the future period from financial publications
E= Semi-annual rate for the future period (D/2)
F= Notional principal from swap contract
G= Discount factor=1/[(forward rate for period 1)(forward rate for period 2)...(forward rate for period t)]
H= PV of notional principal [F × (C/360) × G]

Step 3 – Calculate Swap Rate

Using the results from Steps 1 and 2 above, solve for the theoretical Swap Rate:

$$\text{Theoretical Swap Rate} = \frac{\$12{,}816{,}663}{\$278{,}145{,}000} = 4.61\%$$

Based on the above example, the issuer (fixed-rate payer) will be willing to pay a fixed 4.61 percent rate for the life of the swap contact in return for receiving 6-month LIBOR.

Step 4 - Calculate Swap Spread

With a known Swap Rate, the counterparties can now determine the "swap spread."[4] The market convention is to use a U.S. Treasury security of comparable maturity as a benchmark. For example, if a three-year U.S. Treasury note had a yield to maturity of 4.31 percent, the swap spread in this case would be 30 basis points (4.61% - 4.31% = 0.30%).

Swap Pricing in Practice

The interest rate swap market is large and efficient. While understanding the theoretical underpinnings from which swap rates are derived is important to the issuer, computer programs designed by the major financial institutions and market participants have eliminated the issuer's need to perform complex calculations to determine pricing. Swap pricing exercised in the municipal market is derived from three components: SIFMA percentage (formerly known as the BMA percentage).

[4] The swap spread is the difference between the Swap Rate and the rate offered through other comparable investment instruments with comparable characteristics (e.g., similar maturity).

U.S. Treasury Yield

The choice of the U.S. Treasury yield curve as the risk-free curve is based on the argument that the yields on bonds reflect their credit risk. A bond issued by a government in its own currency is assumed to have no credit risk so that its yield should equal the risk-free rate of interest. Interest rates on U.S. Treasury securities are influenced by market participants' views on a variety of factors including changes to supply and demand for high quality credit relative to the economic cycle, the effect of inflation and investor expectations on interest rate levels, yield curve analysis, and changes in credit spreads between fixed-income quality groups.

LIBOR Spread

LIBOR is the interest rate charged when banks in the London interbank market borrow money from each other. The rate is set for Eurodollar denominated deposits. The LIBOR swap spread is a premium over the risk free rate that the counterparty must pay for the additional credit risk inherent in LIBOR, the current supply/demand relationship for fixed versus floating-rate swaps, and the convenience of holding U.S. Treasury securities.

SIFMA Percentage

The SIFMA index is a tax-exempt, weekly reset index composed of 650 different high-grade, tax-exempt, variable-rate demand obligations (VRDOs). It is a widely used benchmark for borrowers and dealer firms of variable-rate tax-exempt obligations.

The SIFMA percentage is set to approximate average municipal VRDO yields over the long run. In theory, future VRDO rates should equal the after-tax equivalent of LIBOR: [(1-Marginal Tax Rate) × LIBOR] plus a spread to reflect liquidity and other risks. Historically, municipal swaps have used 67 percentage of one-month LIBOR as a benchmark for floating payments in connection with floating-rate transactions. The market uses this percentage based on the historic trading relationship between the LIBOR and the SIFMA index. There are a number of factors that affect the SIFMA percentage and they may manifest themselves during different interest rate environments. The most significant factors influencing the SIFMA percentage are changes in marginal tax laws. Availability of similar substitute investments and the volume of municipal bond issuance also play significant roles in determining the SIFMA percentage during periods of stable rates.

The basic formula for a SIFMA Swap Rate uses a comparable maturity U.S. Treasury yield, adds a LIBOR "swap spread", then multiplies the result by the SIFMA percentage.

SIFMA Swap Rate = [Treasury yield of comparable maturity+ LIBOR Spread] × SIFMA Percentage

Although pricing is generally uniform, it is important to know the components that comprise actual real-life pricing and their effect on valuing the swap at any time during the contract period. Figure 2 below describes the SIFMA Swap Rate calculation.

The Swap Yield Curve

As with most fixed-income investments, there is a positive correlation between time and risk and thus required return. This is also true for swap transactions.

Interest rates tend to vary as a function of maturity. The relationship of interest rates to maturities of specific security types is known as the "yield curve."

Example of 3 Year Generic SIFMA Swap

Current Market Yield to Maturity on a 3 year U.S. Treasury note	**4.31%**
+	
Current 3 year LIBOR swap spread over 3 year U.S Treasury note	**.30%**
=	
3 Year LIBOR Swap Rate	**4.61%**
Multiplied By	
3 year SIFMA percentage	**67%**
=	
3 Year SIFMA Swap Rate	**3.09%**

Figure 2

Swap Yield Curve

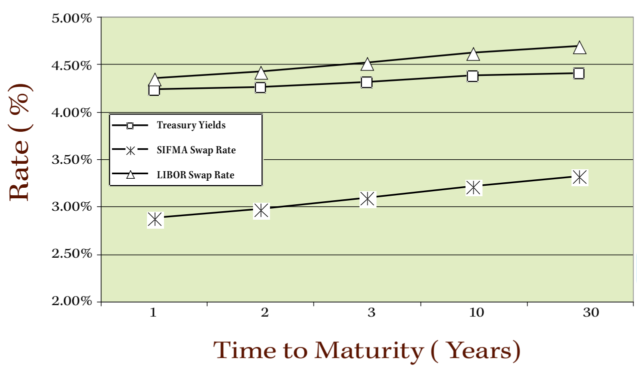


Figure 3

Using the example in Figure 2, Figure 3 graphically displays a hypothetical "swap yield curve" at the time the swap contract was initiated.

For municipal bonds and swaps of similar characteristics, interest rates tend to be higher for longer maturities relative to shorter maturities. At different points in the business cycle, this relationship may be more or less pronounced, causing a more steeply sloped curve or a curve that is relatively flat. In general, the slope of the yield curve reflects investors' expectations about the behavior of interest rates in the future.

Finding the Termination Value of a Swap

Once the swap transaction is completed, changes in market interest rates will change the payments on the floating component of the swap. As discussed in the "Swap Pricing in Theory" section above, at the initiation of an interest rate swap the PV of the floating-rate cash flows minus the PV of the fixed-rate cash flows will be zero at a specific interest rate. If interest rates increase shortly after an interest rate swap has been initiated, the current market expectations are that the future floating-rate payments due under the swap will be higher than those originally expected when the swap was priced. As shown in Figure 4, this benefit will accrue to the fixed-rate payer under the swap and will represent a cost to the floating-rate payer.

If the new cash flows due under the swap are computed and if these are discounted at the appropriate new rate for each future period (i.e., reflecting the current swap yield curve and not the original swap yield curve), the positive PV reflects how the value of the swap to the fixed-rate payer has increased from the initial value of zero and the value of the floating component has declined from the initial zero to a negative amount.

Using the table below, the following example calculates the value of the swap based on a 50 basis point increase in the current SIFMA swap rate. The contract was written for a 3-year, $100,000,000 SIFMA swap that was initiated one year ago. The contract has 2 additional years to run before maturity.

This calculation shows a PV for the swap of $948,617, which reflects the future cash flows discounted at the current market 2-year SIFMA swap rate of 3.59 percent. If the floating-rate payer were to terminate the contract at this point in time, they would be liable to the fixed-rate payer for this amount. Issuers typically construct a "termination matrix" to monitor the exposure they may have based on different interest rate scenarios.

Change in Swap Value to Issuer as Rates Change

Figure 4

	Rates Rise	Rates Fall
Issuer Pays Fixed	+	–
Issuer Receives Fixed	–	+

The counterparties will continuously monitor the market value of their swaps, and if they determine the swap to be a financial burden, they may request to terminate the contract. Significant changes in any of the components (e.g., interest rates, swap spreads, or SIFMA percentage) may cause financial concern for the issuer. It is also important to note that there are other administration fees and/or contractual fees associated with a termination that may influence the decision whether to end the swap.

Notional Amount: $100,000,000				
Existing Fixed Rate Paid by Issuer: 3.09% Current Market Fixed Rate for 2-year SIFMA swap: 3.59%				
year	Annual Fixed Payments @ **3.09%**	Annual Fixed Payments @ **3.59%**	Difference	Present Value
2	$3,090,000	$3,590,000	$ 500,000	$ 482,672
3	$3,090,000	$3,590,000	$ 500,000	$ 465,945
			Swap value	**= $ 948,617**

Swap Pricing Process

The interest rate swap market has evolved from one in which swap brokers acted as intermediaries facilitating the needs of those wanting to enter into interest rate swaps. The broker charged a commission for the transaction but did not participate in the ongoing risks or administration of the swap transaction. The swap parties were responsible for assuring that the transaction was successful.

In the current swap market, the role of the broker has been replaced by a dealer-based market comprised of large commercial and international financial institutions. Unlike brokers, dealers in the over-the-counter market do not charge a commission. Instead, they quote "bid" and "ask" prices at which they stand ready to act as counterparties to their customers in the swap. Because dealers act as middlemen, counterparties need only be concerned with the financial condition of the dealer, and not with the creditworthiness of the other ultimate end user of the swap.

Administrative Conventions

The price of a fixed-to-floating swap is quoted in two parts: a fixed interest rate and an index on which the floating interest rate is based. The floating rate can be based on an index of short-term market rates (such as a given maturity of LIBOR) plus or minus a given margin, or it can be set "flat;" that is, the floating interest rate index itself with no margin added. The convention in the swap market is to quote the fixed interest rate as an "all-in-cost" (AIC), which means that the fixed interest rate is quoted relative to a flat floating-rate index.

The AIC typically is quoted as a spread over U.S. Treasury securities with a maturity corresponding to the term of the swap. For example, a swap dealer might quote a price on a three-year plain vanilla swap at an AIC of "72-76 flat," which means the dealer stands ready to "buy" the swap (that is, enter into the swap as a fixed-rate payer) at 72 basis points over the prevailing three-year interest rate on U.S. Treasuries while receiving floating-rate payments indexed to a specified maturity of LIBOR with no margin, and "sell" (receive a fixed rate and pay the floating rate) if the other party to the swap agrees to pay 76 basis points over U.S. Treasury securities. Bid-ask spreads in the swap market vary greatly depending on the type of agreement. The spread may be less than five basis points for a two- or three-year plain vanilla swap, while spreads for nonstandard, custom-tailored swaps tend to be higher.

Timing of Payments

A swap is negotiated on a "trade date" and takes effect two days later on its initial "settlement date." Interest begins accruing on the "effective date" of the swap, which usually coincides with the initial settlement date. Floating-rate payments are adjusted on periodic "reset dates" based on the prevailing market-determined value of the floating-rate index, with subsequent payments made on a sequence of payment dates (also known as settlement dates) specified by the agreement. Typically, the reset frequency for the floating-rate index is the term of the interest-rate index itself. For example, the floating rate on a plain vanilla swap indexed to the six-month LIBOR would, in most cases, be reset every six months with payment dates following six months later.

Fixed interest payment intervals can be three months, six months, or one year. Semiannual payment intervals are most common because they coincide with the intervals between interest payments on U.S. Treasury bonds. Floating-rate payment intervals need not coincide with fixed-rate payment intervals, although they often do. When payment intervals coincide, it is common practice to exchange only the net difference between the fixed-rate and floating-rate payments.

Conclusion

The goal of this report has been to provide a basic understanding of municipal interest rate swap pricing and to offer the reader a foundation to ask relevant pricing questions to his/her financial advisor or underwriter prior to entering into an interest rate swap.

Pricing municipal interest rate swaps is a multi-faceted exercise incorporating economic, market, tax, and credit variables to determine a fair and appropriate rate. As the market has evolved, pricing transparency has increased, which allows the issuer to use many analytical tools to determine a fair initial and termination price for their interest rate swap(s).

As shown above, small changes in the components that determine interest rate swap pricing can have a financial effect on the issuer. Also, administering a swaps program can be time consuming and requires the issuer to dedicate resources to the analysis and monitoring of the contract.

If an issuer is contemplating entering into a swap transaction, these issues and others should be evaluated in the context of their overall financial plan. The issuer should be able to identify risks inherent in swaps, recognize other alternative financing methods, and avoid using swaps for speculative purposes.

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UNION LABEL OSP 07 101009

California Debt and Investment
Advisory Commission

915 Capitol Mall, Room 400
Sacramento, CA 95814

phone | 916.653.3269
fax | 916.654.7440

cdiac@treasurer.ca.gov
www.treasurer.ca.gov/cdiac