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CALIFORNIA DEBT AND INVESTMENT ADVISORY COMMISSION

Group Exercise & Discussion: Bond Math

February 2019

KNN public finance

KNN Public Finance

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- KNN Public Finance is an employee-owned independent municipal advisory firm
- Headquartered in Oakland, with additional offices in Los Angeles and Newport Beach
- All advisors are registered with the MSRB, with Series 50 licenses

Joanna Bowes
Managing Director, Partner

- Manager of KNN Education Group
- 23 plus years of experience in public finance
  - Financial Advisor
  - Underwriter
  - Investment Banker
- Expertise with complex financing issues
- MBA, University of Connecticut; BA, Northwestern University

Erwin Tam
Vice President

- Quantitative lead for KNN Education Group
- 14 plus years of experience in public finance
  - Financial Advisor
  - Investment Banker
  - Public-Private Partnerships
- Prior experience with RBC Capital Markets, Bear, Stearns & Co. Inc., and PFM
- BA, University of California, Berkeley

Why is Bond Math Important?

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- Understand the terms and concepts of debt service on bonds that you have issued
  - General obligation
  - Revenue Bond
    - Water
    - Power
    - Judgement
- Understand the numbers behind debt service
- How to use excel to manage bond payments

General Bond Terminology

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▪ Principal or Par	Total amount borrowed
▪ Coupon	Interest due to the investor, typically paid semiannual
▪ Yield	Rate of return to the investor
▪ Price	The price an investor will pay to receive the yield
▪ Maturity	Date at which principal is due to the bondholder
▪ Dated Date	Date from which an investor is entitled to receive interest
▪ Delivery Date	Settlement date of the bond (closing date for primary bond issuance)
▪ Debt Service	Total principal and interest payments on bond
▪ Call Date	Redemption date of a bond prior to maturity at the option of the issuer
▪ Call Premium	Dollar amount over 100% which is paid to the investor when bonds are called

Bond Pricing Terminology

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- **Par Bond**
  - Coupon and Yield are equal
  - Price equal to 100.000
  - Every $1,000 of bonds issued will produce $1,000 in proceeds

- **Premium Bond**
  - Coupon is greater than Yield
  - Price greater than 100.000
  - Every $1,000 of bonds issued will produce over $1,000 in proceeds
  - Similar to receiving points in a mortgage

- **Discount Bond**
  - Coupon is less than Yield
  - Price less than 100.000
  - Every $1,000 of bonds issued will produce less than $1,000 in proceeds
  - Capital Appreciation Bonds are Discount Bonds
  - Similar to paying points in a mortgage

Time Value of Money

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Issuer		Investor
Gets money upfront	←	Loans money upfront
Repays interest and principal borrowed over time	→	Gets interest and principal over time

Bond Pricing

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Par Bonds

- If coupon and yield are the same, the price of the bond is 100.000

Premium Callable Bonds

- Bond price needs to be calculated assuming bonds are redeemed on the call date and at maturity
- Whatever results in the lower bond price, is the price of the bond
  - A callable (at par) premium bond will always have its lowest price at the call date
  - For a theoretical 20 year bond, 5% coupon and 4% yield with a 10 year par call:

Bond Price to Call Date = 108.175

Bond Price to Maturity = 113.677

Bond Price Rounding

- Prices are shown as **truncated** to the 3rd decimal

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Bond Pricing Formula in Excel

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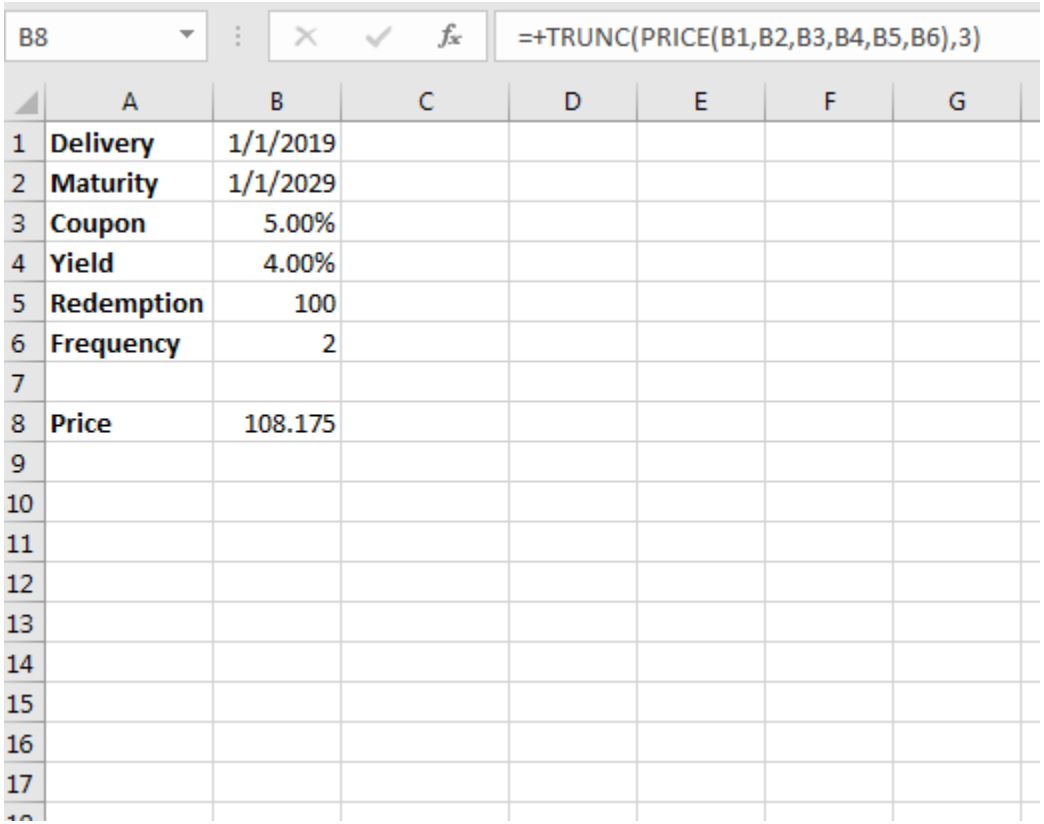
- Excel's Price Function has 6 components
  - Settlement [Delivery Date]
  - Maturity [Maturity]
  - Rate [Coupon]
  - Yld [Yield]
  - Redemption [Call Premium]
  - Frequency [Semi-annual, 2]

Bond Price Formula: Excel Example

- What is the bond price of the a bond assuming the following?
  - 10 Year Maturity
  - 5.00% Coupon
  - 4.00% Yield



	A	B	C	D	E	F	G
1	**Delivery**	1/1/2019					
2	**Maturity**	1/1/2029					
3	**Coupon**	5.00%					
4	**Yield**	4.00%					
5	**Redemption**	100					
6	**Frequency**	2					
7							
8	**Price**	108.175					
9							
10							
11							
12							
13							
14							
15							
16							
17							

Bond Price and Bond Proceeds

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An issuer wants $1,000,000 new proceeds for a project. How much does it need to issue to receive that amount, assuming the following bond prices:

- 100.000 (par bond)

$$\frac{\$1{,}000{,}000}{(100 / 100)} = \$1{,}000{,}000$$

Par bonds result in the same amount of bonds and proceeds

- 105.000 (premium bond)

$$\frac{\$1{,}000{,}000}{(105 / 100)} = \$952{,}381$$

Premium bonds require less bonds to be issued to receive the same proceeds

- 95.000 (discount bond)

$$\frac{\$1{,}000{,}000}{(95 / 100)} = \$1{,}052{,}632$$

Discount bonds require more bonds to be issued to receive the same proceeds

Changes in Yield and Bond Price

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- When yield changes, bond price changes:

Yield Free

Coupon	Yield	Price
0.00%	0.00%	100.000

Increase in Yield

Coupon	Yield	Price
0.00%	2.00% ▲	98.029 ▼

- Yield and Price are inversely related
  - As yields increase, price of a fixed-rate bond decreases
  - As yields decrease, price of a fixed rate bond increase

Debt Service

- **Total Debt Service = Principal + (Principal** x **# of semi-annual periods** x **½ of coupon)**

- Total Debt Service is the sum of all principal and interest payments over time for a bond
- Fixed-rate municipal bonds pay interest semi-annual (2 times a year)
- Principal is paid at the maturity of the Bonds

- Calculating total debt service for a bond with:
  - $1,000,000 principal
  - 5.0% coupon
  - 5 year term

Debt Service Example

- Calculating total debt service for a bond with:
  - $1,000,000 principal, 5.0% coupon and a 5 year term
- **Total Debt Service = Principal + (Principal** x **# of semi-annual periods** x **½ of coupon)**

$1,000,000 + ($1,000,000 x 10 x ½ of 5.0%)

$1,000,000 + ($1,000,000 x 10 x 2.5%)

$1,000,000 + ($1,000,000 x .25)

$1,000,000 + $250,000

$1,250,000

Date	Principal	Coupon	Interest	Total Debt Service
7/1/2019				
1/1/2020			25,000	25,000
7/1/2020			25,000	25,000
1/1/2021			25,000	25,000
7/1/2021			25,000	25,000
1/1/2022			25,000	25,000
7/1/2022			25,000	25,000
1/1/2023			25,000	25,000
7/1/2023			25,000	25,000
1/1/2024			25,000	25,000
7/1/2024	1,000,000	5.00%	25,000	1,025,000
Total	1,000,000		250,000	1,250,000

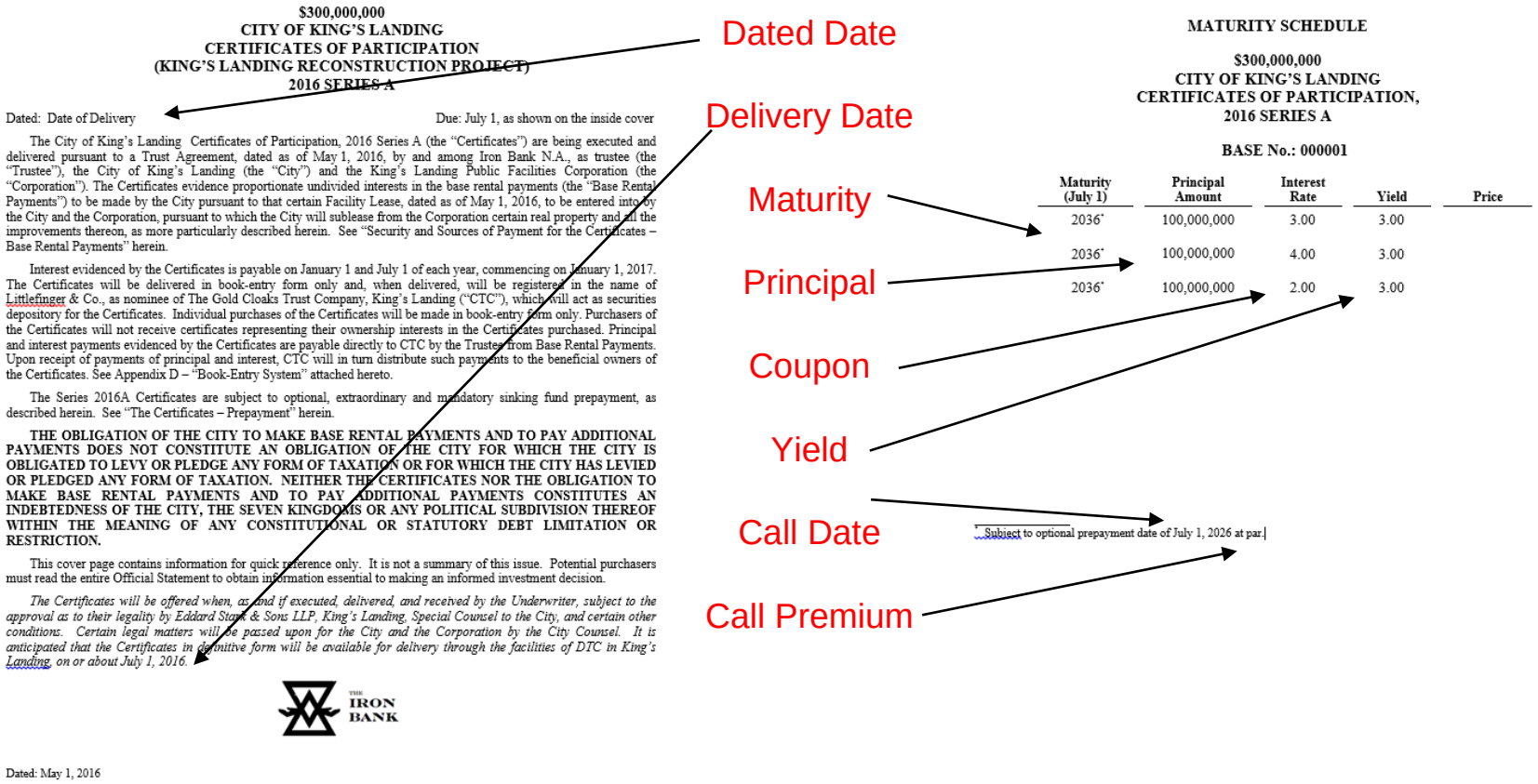
Group Activity: Introduction

$300,000,000
CITY OF KING'S LANDING
CERTIFICATES OF PARTICIPATION
(KING'S LANDING RECONSTRUCTION PROJECT)
2016 SERIES A

- The City of King's Landing is issuing $300 million in Certificates of Participation (COPs) to fund capital improvement projects
- The City needs your help in explaining bond pricing

Question 1: Review

For the each of the bonds, connect the terminology below with its location in the official statement



$300,000,000
CITY OF KING'S LANDING
CERTIFICATES OF PARTICIPATION
(KING'S LANDING RECONSTRUCTION PROJECT)
2016 SERIES A

Dated: Date of Delivery

Due: July 1, as shown on the inside cover

The City of King's Landing Certificates of Participation, 2016 Series A (the "Certificates") are being executed and delivered pursuant to a Trust Agreement, dated as of May 1, 2016, by and among Iron Bank N.A., as trustee (the "Trustee"), the City of King's Landing (the "City") and the King's Landing Public Facilities Corporation (the "Corporation"). The Certificates evidence proportionate undivided interests in the base rental payments (the "Base Rental Payments") to be made by the City pursuant to that certain Facility Lease, dated as of May 1, 2016, to be entered into by the City and the Corporation, pursuant to which the City will sublease from the Corporation certain real property and all the improvements thereon, as more particularly described herein. See "Security and Sources of Payment for the Certificates – Base Rental Payments" herein.

Interest evidenced by the Certificates is payable on January 1 and July 1 of each year, commencing on January 1, 2017. The Certificates will be delivered in book-entry form only and, when delivered, will be registered in the name of Littlefinger & Co., as nominee of The Gold Cloaks Trust Company, King's Landing ("CTC"), which will act as securities depository for the Certificates. Individual purchases of the Certificates will be made in book-entry form only. Purchasers of the Certificates will not receive certificates representing their ownership interests in the Certificates purchased. Principal and interest payments evidenced by the Certificates are payable directly to CTC by the Trustee from Base Rental Payments. Upon receipt of payments of principal and interest, CTC will in turn distribute such payments to the beneficial owners of the Certificates. See Appendix D – "Book-Entry System" attached hereto.

The Series 2016A Certificates are subject to optional, extraordinary and mandatory sinking fund prepayment, as described herein. See "The Certificates – Prepayment" herein.

THE OBLIGATION OF THE CITY TO MAKE BASE RENTAL PAYMENTS AND TO PAY ADDITIONAL PAYMENTS DOES NOT CONSTITUTE AN OBLIGATION OF THE CITY FOR WHICH THE CITY IS OBLIGATED TO LEVY OR PLEDGE ANY FORM OF TAXATION OR FOR WHICH THE CITY HAS LEVIED OR PLEDGED ANY FORM OF TAXATION. NEITHER THE CERTIFICATES NOR THE OBLIGATION TO MAKE BASE RENTAL PAYMENTS AND TO PAY ADDITIONAL PAYMENTS CONSTITUTES AN INDEBTEDNESS OF THE CITY, THE SEVEN KINGDOMS OR ANY POLITICAL SUBDIVISION THEREOF WITHIN THE MEANING OF ANY CONSTITUTIONAL OR STATUTORY DEBT LIMITATION OR RESTRICTION.

This cover page contains information for quick reference only. It is not a summary of this issue. Potential purchasers must read the entire Official Statement to obtain information essential to making an informed investment decision.

The Certificates will be offered when, as and if executed, delivered, and received by the Underwriter, subject to the approval as to their legality by Eddard Stark & Sons LLP, King's Landing, Special Counsel to the City, and certain other conditions. Certain legal matters will be passed upon for the City and the Corporation by the City Counsel. It is anticipated that the Certificates in definitive form will be available for delivery through the facilities of DTC in King's Landing, on or about July 1, 2016.

THE IRON BANK

Dated: May 1, 2016

Dated Date

Delivery Date

Maturity

Principal

Coupon

Yield

Call Date

Call Premium

MATURITY SCHEDULE

$300,000,000
CITY OF KING'S LANDING
CERTIFICATES OF PARTICIPATION,
2016 SERIES A

BASE No.: 000001

Maturity (July 1)	Principal Amount	Interest Rate	Yield	Price
2036*	100,000,000	3.00	3.00	
2036*	100,000,000	4.00	3.00	
2036*	100,000,000	2.00	3.00	

* Subject to optional prepayment date of July 1, 2026 at par.

Question 2: Review

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For the each of the bonds, assume a 20-year maturity and identify the bond price and the terminology describing the type of bond

Coupon	Yield	Price	Terminology
3.0%	3.0%	100.000	Par
4.0%	3.0%	114.957	Premium
2.0%	3.0%	85.042	Discount

Question 3: Review

CDIAC

For the each of the bonds, assume a 20-year maturity and identify the bond price and the terminology describing the type of bond

Coupon	Yield	Price	Terminology
2.0%	2.0%	100.000	Par
3.0%	2.0%	116.417	Premium
4.0%	2.0%	132.834	Premium
2.0%	4.0%	72.644	Discount
3.0%	4.0%	86.322	Discount
4.0%	4.0%	100.000	Par

Question 4: Review

Based on your answers above fill in the following:

When yields increase, bond price **decreases**.

When yields decrease, bond price **increases**.

Example from question 3:

Coupon	Yield	Price
2.0%	2.0%	100.000
2.0%	4.0%	72.644

Question 5: Review

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The 20-year bond in question 2 was non-callable.

- Which bond will have a difference in price with a 10-year par call? **C**
- Based on your previous answer, will the bond price be higher or lower with a 10-year par call as compared to a non-callable bond? **Lower**

Bond	Coupon	Yield	Non-Call Price	Callable Bond Price
A	2.0%	3.0%	85.042	85.042
B	3.0%	3.0%	100.000	100.000
C	**4.0%**	**3.0%**	**114.957**	**108.584**

Question 6: Review

Based on the following bonds, what would be the principal amount of bonds needed to generate $100 million in proceeds for each bond? (Round up to $5,000)

Coupon	Yield	Price	Principal
4.0%	5.0%	92.205	**108,455,000**
5.0%	5.0%	100.000	**100,000,000**
6.0%	5.0%	107.794	**92,770,000**

$$\text{Principal Amount Required} = \frac{\text{New Proceeds}}{(\text{Price}/100)}$$

$$\$108{,}453{,}988 = \frac{\$100{,}000{,}000}{(92.205/100)}$$

Question 7: Review

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Based on the bond size you calculated for question 6, what would be the total debt service (principal & interest) assuming a 10 year term for the bond?

Coupon	Yield	Principal (from question 6)	Interest	Total Debt Service
4.0%	5.0%	108,455,000	43,382,000	151,837,000
5.0%	5.0%	100,000,000	50,000,000	150,000,000
6.0%	5.0%	92,770,000	55,662,000	148,432,000

Total Debt Service = Principal + (Principal x # of semi-annual periods x ½ of coupon)

Total Debt Service = 108,455,000 + (108,455,000 x 20 x 2%)

Total Debt Service = 108,455,000 + 43,382,000

Total Debt Service = 151,837,000

Questions & Answers

Appendix:
Detailed Bond Price

Bond Price Formula

- The dollar price of a bond is the present value of the future cashflows at the market yield
- Coupon, yield, and time are the only factors in price
- There are several methods to derive bond price, including:

$$\text{Bond Dollar Price} = \frac{100}{\left(1 + \frac{\text{Yield}}{2}\right)^{\text{\# of semi-annual periods}}} + 100 \times \frac{\text{Coupon}}{2} \times \frac{1 - \left(1 + \frac{\text{Yield}}{2}\right)^{-\text{\# of semi-annual periods}}}{\left(\frac{\text{Yield}}{2}\right)}$$

Present value of the principal at maturity

Present value of the interest payments over time

Bond Price Formula: Example

- What is the price of a municipal bond assuming:
  - 10 Year Maturity
  - 5.00% Coupon
  - 4.00% Yield

$$\textbf{Bond Price} = \frac{100}{\left(1 + \frac{\text{Yield}}{2}\right)^{\text{\# of semi-annual periods}}} + 100 \times \frac{\text{Coupon}}{2} \times \frac{1 - \left(1 + \frac{\text{Yield}}{2}\right)^{-\text{\# of semi-annual periods}}}{\left(\frac{\text{Yield}}{2}\right)}$$

Bond Price Formula: Example

- What is the price of a municipal bond assuming:
  - 10 Year Maturity (20 semi-annual periods)
  - 5.00% Coupon
  - 4.00% Yield

$$\text{Bond Price} = \frac{100}{\left(1 + \frac{\text{Yield}}{2}\right)^{20}} + 100 \times \frac{\text{Coupon}}{2} \times \frac{1 - \left(1 + \frac{\text{Yield}}{2}\right)^{-20}}{\left(\frac{\text{Yield}}{2}\right)}$$

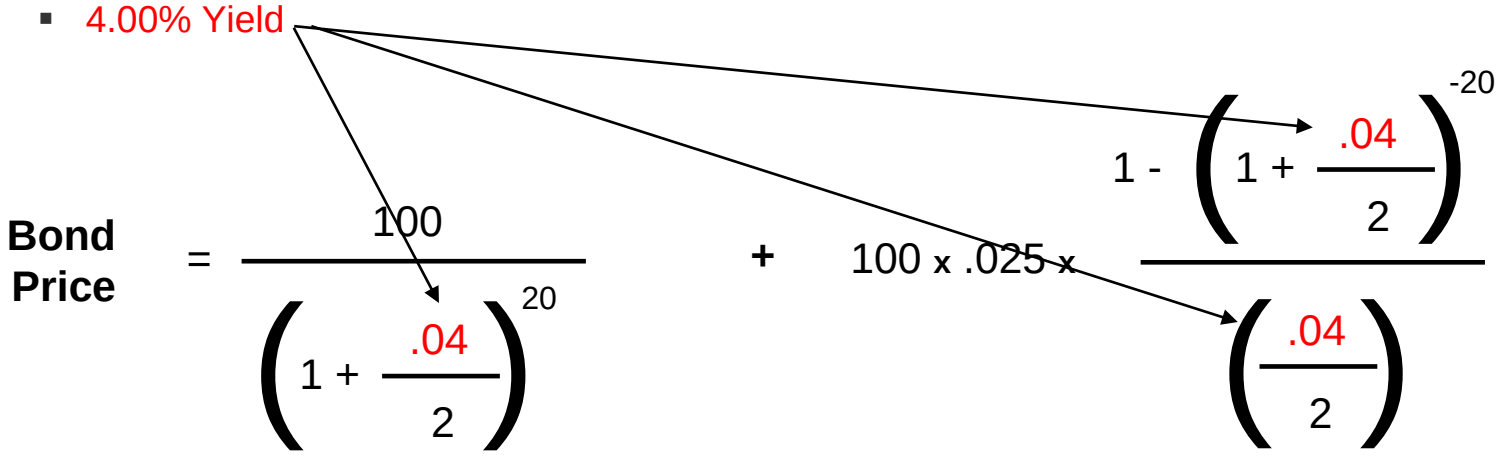
Bond Price Formula: Example

- What is the price of a municipal bond assuming:
  - 10 Year Maturity (20 semi-annual periods)
  - 5.00% Coupon
  - 4.00% Yield

$$\text{Bond Price} = \frac{100}{\left(1 + \frac{\text{Yield}}{2}\right)^{20}} + 100 \times .025 \times \frac{1 - \left(1 + \frac{\text{Yield}}{2}\right)^{-20}}{\left(\frac{\text{Yield}}{2}\right)}$$

Bond Price Formula: Example

- What is the price of a municipal bond assuming:
  - 10 Year Maturity (20 semi-annual periods)
  - 5.00% Coupon
  - 4.00% Yield



Bond Pricing Formula: Example

- Municipal bond convention for pricing is truncation at the 3rd decimal
- No rounding!

Bond Price = 108.17571~~66~~

MSRB Rule G-42 Disclosures

MSRB Rule G-42: Disclosure of Conflicts of Interest and Legal or Disciplinary Events

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Pursuant to Municipal Securities Rulemaking Board ("MSRB") Rule G-42, on Duties of Non-Solicitor Municipal Advisors, Municipal Advisors are required to make certain written disclosures to clients which include, amongst other things, Conflicts of Interest and any Legal or Disciplinary events of KNN Public Finance, LLC ("KNN Public Finance") and its associated persons.

Conflicts of Interest

KNN Public Finance represents that in connection with the issuance of municipal securities, KNN Public Finance may receive compensation from an Issuer or Obligated Person for services rendered, which compensation is contingent upon the successful closing of a transaction and/or is based on the size of a transaction. Consistent with the requirements of MSRB Rule G-42, KNN Public Finance hereby discloses that such contingent and/or transactional compensation may present a potential conflict of interest regarding KNN Public Finance's ability to provide unbiased advice to enter into such transaction. This conflict of interest will not impair KNN Public Finance's ability to render unbiased and competent advice or to fulfill its fiduciary duty to the Issuer.

If KNN Public Finance becomes aware of any additional potential or actual conflict of interest after this disclosure, KNN Public Finance will disclose the detailed information in writing to the Issuer in a timely manner.

Legal or Disciplinary Events

KNN Public Finance, LLC, has never been subject to any legal, disciplinary or regulatory actions nor was it ever subject to any legal, disciplinary or regulatory actions previously, when it was a division of Zions First National Bank or Zions Public Finance, Inc.

A regulatory action disclosure has been made on Form MA-I for one of KNN Public Finance municipal advisory personnel relating to a 1998 U.S. Securities and Exchange Commission ("SEC") order that was filed while the municipal advisor was employed with a prior firm, (not KNN Public Finance). The details of which are available in Item 9; C(1), C(2), C(4), C(5) and the corresponding regulatory action DRP section on Form MA and Item 6C; (1), (2), (4), (5) and the corresponding regulatory action DRP section on Form MA-I. Issuers may electronically access KNN Public Finance's most recent Form MA and each most recent Form MA-I filed with the Commission at the following website: www.sec.gov/edgar/searchedgar/companysearch.html.

The SEC permits certain items of information required on Form MA and Form MA-I to be provided by reference to such required information already filed on a regulatory system (e.g., FINRA CRD). The above noted regulatory action has been referenced on both Form MA and MA-I due to the information already filed on FINRA's CRD system and is publicly accessible through BrokerCheck at http://brokercheck.finra.org. For purposes of accessing such BrokerCheck information, the Municipal Advisor's CRD number is 4457537.