

# Bond Math: A Deeper Dive

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# KNN Public Finance, LLC

- KNN Public Finance, LLC is an employee-owned independent municipal advisory firm.
  - Registered with the SEC and MSRB.
- Staff of 18, with offices in the Bay Area and Los Angeles.
- SEC rules assign a fiduciary duty to the municipal advisor – the highest standard of care; by law, we must put our clients' interest ahead of our own.



# Why Bond Math is Important

- Management of existing debt portfolio.
- Priorities for new debt issuance.
- Understand impact of movements in the market upon debt.



# Agenda

- Overview of Basic Bond Math Concepts
- Case Study – Applying Bond Math Concepts
- Impact of Recent Market Movements
- Frequently Asked Questions
- Audience Q&A



# Section 1: Overview of Basic Bond Math Concepts

# General Bond Terminology

- Principal or Par Amount      Stated amount borrowed via a loan
- Maturity                              Date at which principal is due to the bondholder, typically paid annually
- Interest / Coupon Rate      Interest due to the investor, typically paid semiannual
- Dollar Price                              The price an investor will pay to receive the yield
- Yield                                      Rate of return to the investor based on price paid on investment
- CUSIP Number                      Unique identification number assigned to registered bonds

## Sample from Inside Cover of Official Statement

<i>Maturity (June 1)</i>	<i>Principal Amount</i>	<i>Interest Rate</i>	<i>Yield</i>	<i>Price</i>	<i>CUSIP No.<sup>†</sup></i>
2023	\$ 640,000	5.000%	1.630%	102.619	587657EX1
2024	640,000	5.000	1.750	105.687	587657EY9
2025	675,000	5.000	1.810	108.626	587657EZ6



# General Bond Terminology (continued)

- Dated Date                      Date from which an investor is entitled to receive interest
- Delivery Date                  Settlement date of the bond (closing date for primary bond issuance)
- Yield to Maturity              Rate of return to the investor if the investment is held to maturity
- Call Date                        Redemption date of a bond prior to maturity at the option of the issuer
- Call Premium                  Any amount over 100% which is paid to the investor when bonds are called
- Basis Point                      1/100 of 1%
- Serial Bond                      Bond with single maturity
- Term Bond                        Bond with sinking fund principal payments over multiple years
- Amortization                  The shape of principal repayment of a loan



# Bond Statistics Terminology

- True Interest Cost (T.I.C.) Blended cost of borrowing that factors in time value of money
- All-in T.I.C. Blended cost of borrowing that factors in time value of money AND costs of issuance
- Net Interest Cost (N.I.C.) Blended cost of borrowing that factors in the average interest rate weighted for the time to maturity and does NOT factor in the time value of money
- Arbitrage Yield Maximum rate that tax exempt bond proceeds can earn





# Bond Pricing Terminology

## Par Bond

- Coupon and Yield are equal
- Price equal to 100.000
- Every \$1,000 of bonds issued will produce exactly \$1,000 in proceeds, before deduction of underwriter's discount

## Premium Bond

- Coupon is greater than Yield
- Price greater than 100.000
- Every \$1,000 of bonds issued will produce over \$1,000 in proceeds

## Discount Bond

- Coupon is less than Yield
- Price less than 100.000
- Every \$1,000 of bonds issued will produce less than \$1,000 in proceeds



# Bond Pricing Methodology

## Par Bonds

- If coupon and yield are the same, the price of the bond is 100.000.

## Premium Callable Bonds

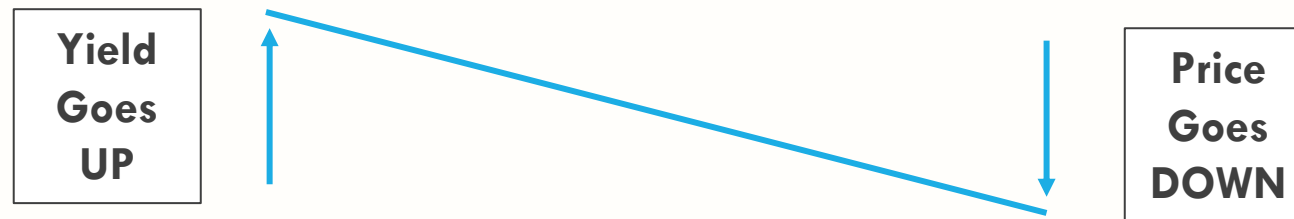
- Bond price must be calculated utilizing the lower of the yield (to call) versus the yield to maturity.
- For premium callable bonds, the yield to call is lower than the yield to maturity.

## Bond Price Rounding

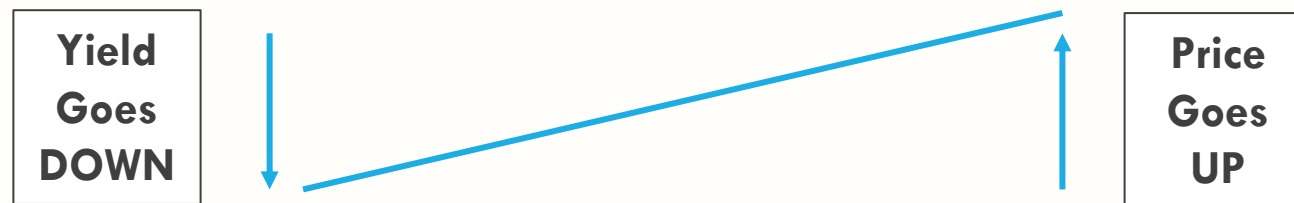
- Prices are shown as **truncated** to the 3<sup>rd</sup> decimal place.

# Changes in Yield and Bond Price

- Yield and Price are inversely related
- For fixed rate bonds that have already priced, as market yields increase, the dollar price of the fixed-rate bond decreases:



- As yields decrease, price of a fixed rate bond increase





## Section 2: Case Study – Applying Bond Math Concepts



# Case Study: Summary of Transaction

- New money Certificates of Participation to fund a new California County jail facility.
- Borrowing term of 25 years.
- Structured with level fiscal year payments.
- Funded costs of issuance, including bond insurance and surety bond policy.
- 10-year par optional call provision.
- Sold via competitive method of sale.

# Sources and Uses

- Par plus premium equals Total Sources.
- Sum of all costs, including project cost, equals Total Uses.
- Total Sources equals Total Uses.

<b>Sources and Uses</b>	
<b>Sources</b>	
Par Amount:	\$28,975,000
Premium:	<u>3,619,644</u>
<b>Total Sources:</b>	<b>\$32,594,644</b>
<b>Uses:</b>	
Project Fund:	\$32,000,000
Cost of Issuance:	285,000
Underwriter's Discount:	123,992
Bond Insurance Premium:	138,158
Surety Bond Premium:	43,702
Rounding:	<u>3,792</u>
<b>Total Uses:</b>	<b>\$32,594,644</b>

# Bond Pricing Report

- Pricing consisted of all serial bonds.
- All maturities were priced at a premium structure, where the coupon is higher than the yield, except the 2046 maturity, which priced at a discount.
- Premium bonds that are subject to optional call are priced assuming the bonds are called.
  - We also show the YTM, which is the rate of return the investor receives if the bonds are NOT called.
- For discount bonds, the “yield” is the YTM.

Bond Pricing Report					
Date	Principal Maturity	Coupon / Interest Rate	Yield	Yield to Maturity	Dollar Price
6/1/2023	\$640,000	5.00%	1.63%		102.619
6/1/2024	640,000	5.00%	1.75%		105.687
6/1/2025	675,000	5.00%	1.81%		108.626
6/1/2026	705,000	5.00%	1.91%		111.231
6/1/2027	740,000	5.00%	1.92%		114.017
6/1/2028	780,000	5.00%	2.09%		115.778
6/1/2029	820,000	5.00%	2.20%		117.558
6/1/2030	860,000	5.00%	2.27%		119.379
6/1/2031	900,000	5.00%	2.35%		120.922
6/1/2032	945,000	5.00%	2.42%		122.357
6/1/2033	995,000	5.00%	2.54%	2.72%	121.192 c
6/1/2034	1,045,000	5.00%	2.65%	2.96%	120.136 c
6/1/2035	1,095,000	5.00%	2.73%	3.15%	119.375 c
6/1/2036	1,150,000	5.00%	2.80%	3.30%	118.713 c
6/1/2037	1,210,000	5.00%	2.88%	3.44%	117.963 c
6/1/2038	1,270,000	5.00%	2.97%	3.57%	117.125 c
6/1/2039	1,330,000	5.00%	3.03%	3.67%	116.570 c
6/1/2040	1,400,000	5.00%	3.10%	3.76%	115.927 c
6/1/2041	1,470,000	5.00%	3.19%	3.86%	115.107 c
6/1/2042	1,540,000	5.00%	3.21%	3.91%	114.926 c
6/1/2043	1,620,000	4.00%	3.60%	3.77%	103.270 c
6/1/2044	1,685,000	4.00%	3.70%	3.83%	102.439 c
6/1/2045	1,750,000	4.00%	3.80%	3.89%	101.617 c
6/1/2046	1,820,000	3.75%	3.90%		97.684
6/1/2047	1,890,000	5.00%	3.50%	4.20%	112.333 c
Total:	\$28,975,000				

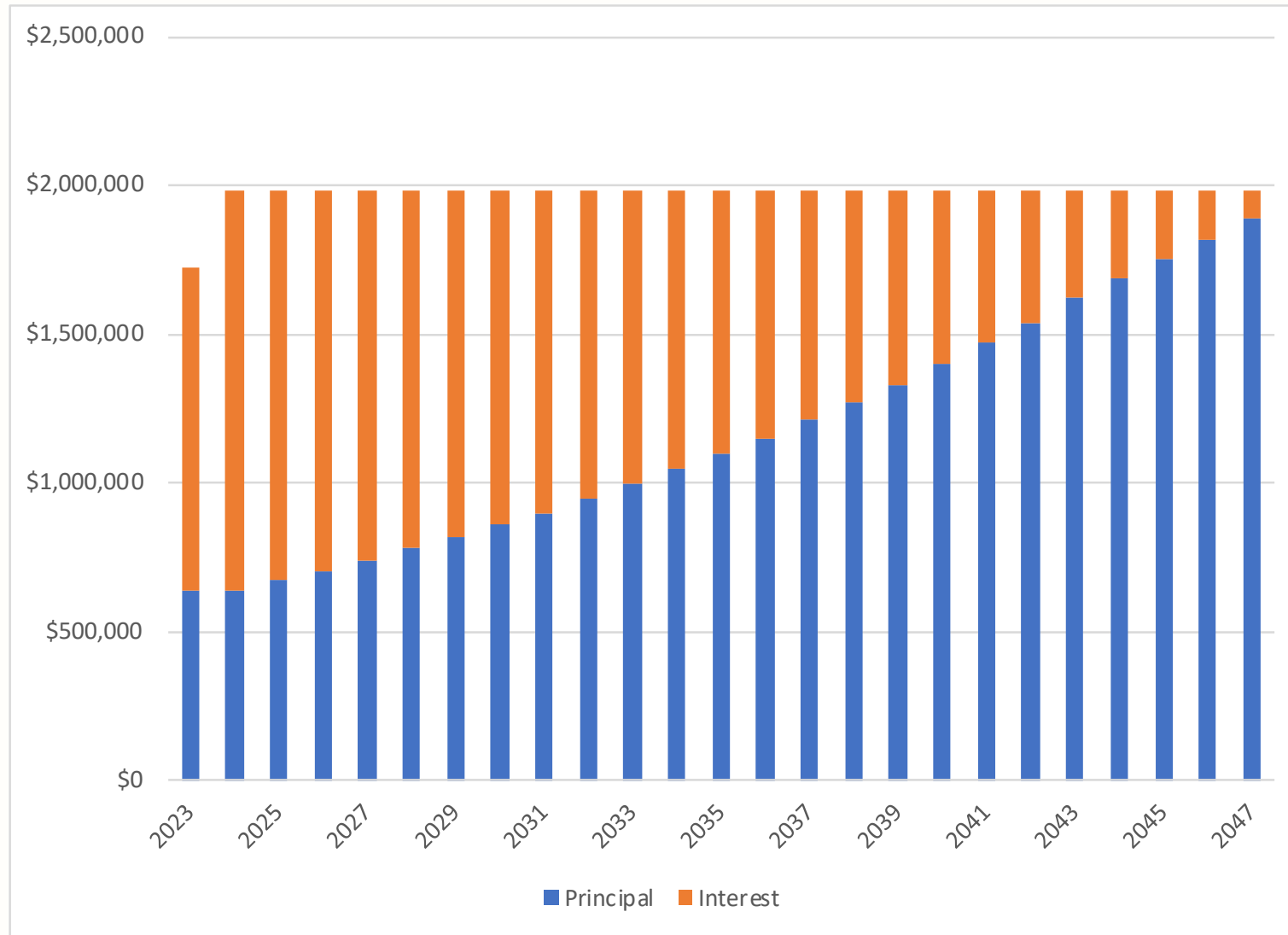
# Debt Service and Amortization

- The principal amortization schedule created a level overall payment structure, by fiscal year, similar to monthly mortgage or car loan payment.
  - In this example, the short first period resulted in slightly less payment in first year.
- Repayment of principal increases over time while payment of interest decreases over time.

Debt Service Schedule			
FY Ending	Principal Amount	Interest Cost	Total Cost
2023	\$640,000	\$1,081,257	\$1,721,257
2024	640,000	1,343,450	1,983,450
2025	675,000	1,311,450	1,986,450
2026	705,000	1,277,700	1,982,700
2027	740,000	1,242,450	1,982,450
2028	780,000	1,205,450	1,985,450
2029	820,000	1,166,450	1,986,450
2030	860,000	1,125,450	1,985,450
2031	900,000	1,082,450	1,982,450
2032	945,000	1,037,450	1,982,450
2033	995,000	990,200	1,985,200
2034	1,045,000	940,450	1,985,450
2035	1,095,000	888,200	1,983,200
2036	1,150,000	833,450	1,983,450
2037	1,210,000	775,950	1,985,950
2038	1,270,000	715,450	1,985,450
2039	1,330,000	651,950	1,981,950
2040	1,400,000	585,450	1,985,450
2041	1,470,000	515,450	1,985,450
2042	1,540,000	441,950	1,981,950
2043	1,620,000	364,950	1,984,950
2044	1,685,000	300,150	1,985,150
2045	1,750,000	232,750	1,982,750
2046	1,820,000	162,750	1,982,750
2047	1,890,000	94,500	1,984,500
Total:	\$28,975,000	\$20,367,157	\$49,342,157



# Debt Service and Amortization





# Calculation of True Interest Cost (T.I.C.)

The T.I.C. is the present value rate applied to the future stream of payments that results in the purchase price.

## Step #1: Calculate Purchase Price

Bond Par Amount:	\$28,975,000.00
+ Bond Premium:	3,619,643.90
- Underwriter's Discount:	<u>123,991.85</u>
Purchase Price:	<u>\$32,470,652.05</u>

## Step #2: Calculate Present Value

Payment Date	Payment	Present Value	Payment Date	Payment	Present Value
8/18/2022	0.00	0.00	12/1/2034	444,100.00	287,582.12
12/1/2022	393,531.53	389,569.34	6/1/2035	1,539,100.00	979,191.81
6/1/2023	1,327,725.00	1,291,317.97	12/1/2035	416,725.00	260,477.56
12/1/2023	671,725.00	641,854.22	6/1/2036	1,566,725.00	962,129.13
6/1/2024	1,311,725.00	1,231,423.72	12/1/2036	387,975.00	234,079.92
12/1/2024	655,725.00	604,792.35	6/1/2037	1,597,975.00	947,218.62
6/1/2025	1,330,725.00	1,205,848.34	12/1/2037	357,725.00	208,328.83
12/1/2025	638,850.00	568,752.22	6/1/2038	1,627,725.00	931,324.31
6/1/2026	1,343,850.00	1,175,424.74	12/1/2038	325,975.00	183,241.58
12/1/2026	621,225.00	533,842.06	6/1/2039	1,655,975.00	914,562.39
6/1/2027	1,361,225.00	1,149,247.60	12/1/2039	292,725.00	158,832.44
12/1/2027	602,725.00	499,945.57	6/1/2040	1,692,725.00	902,372.03
6/1/2028	1,382,725.00	1,126,831.97	12/1/2040	257,725.00	134,981.92
12/1/2028	583,225.00	466,959.59	6/1/2041	1,727,725.00	889,024.01
6/1/2029	1,403,225.00	1,103,799.80	12/1/2041	220,975.00	111,712.52
12/1/2029	562,725.00	434,889.62	6/1/2042	1,760,975.00	874,644.80
6/1/2030	1,422,725.00	1,080,248.37	12/1/2042	182,475.00	89,043.40
12/1/2030	541,225.00	403,738.66	6/1/2043	1,802,475.00	864,146.62
6/1/2031	1,441,225.00	1,056,267.94	12/1/2043	150,075.00	70,688.11
12/1/2031	518,725.00	373,507.49	6/1/2044	1,835,075.00	849,203.27
6/1/2032	1,463,725.00	1,035,479.39	12/1/2044	116,375.00	52,909.95
12/1/2032	495,100.00	344,107.96	6/1/2045	1,866,375.00	833,674.28
6/1/2033	1,490,100.00	1,017,506.14	12/1/2045	81,375.00	35,711.52
12/1/2033	470,225.00	315,462.08	6/1/2046	1,901,375.00	819,794.37
6/1/2034	1,515,225.00	998,707.72	12/1/2046	47,250.00	20,015.15
			6/1/2047	1,937,250.00	806,236.54
					<b>\$49,342,156.53</b>
					<b>\$32,470,652.05</b>

## Step #3: Calculate T.I.C.

Total PV:	<b>\$32,470,652.05</b>
True Interest Cost:	<b>3.568308%</b>

# Calculation of All-In T.I.C.

The All-In T.I.C. is the present value rate applied to the future stream of payments that results in the purchase price less cost of issuance.

## Step #1: Calculate Target Value

Bond Par Amount:	\$28,975,000.00
+ Bond Premium:	3,619,643.90
- Underwriter's Discount:	-123,991.85
<b>- Cost of Issuance:</b>	<b>-285,000.00</b>
<b>- Bond Insurance Premium</b>	<b>-138,158.04</b>
<b>- Surety Bond Premium:</b>	<b>-43,701.90</b>
Target Value	\$32,003,792.11

## Step #2: Calculate Present Value

Payment Date	Payment	Present Value	Payment Date	Payment	Present Value
8/18/2022	0.00	0.00	12/1/2034	444,100.00	282,922.58
12/1/2022	393,531.53	389,421.17	6/1/2035	1,539,100.00	962,686.32
6/1/2023	1,327,725.00	1,289,969.01	12/1/2035	416,725.00	255,916.70
12/1/2023	671,725.00	640,757.61	6/1/2036	1,566,725.00	944,654.44
6/1/2024	1,311,725.00	1,228,502.87	12/1/2036	387,975.00	229,675.70
12/1/2024	655,725.00	602,956.86	6/1/2037	1,597,975.00	928,779.05
6/1/2025	1,330,725.00	1,201,389.77	12/1/2037	357,725.00	204,137.52
12/1/2025	638,850.00	566,272.71	6/1/2038	1,627,725.00	911,980.81
6/1/2026	1,343,850.00	1,169,522.67	12/1/2038	325,975.00	179,316.43
12/1/2026	621,225.00	530,808.53	6/1/2039	1,655,975.00	894,377.11
6/1/2027	1,361,225.00	1,141,957.66	12/1/2039	292,725.00	155,223.63
12/1/2027	602,725.00	496,444.16	6/1/2040	1,692,725.00	881,283.30
6/1/2028	1,382,725.00	1,118,196.51	12/1/2040	257,725.00	131,739.74
12/1/2028	583,225.00	463,073.11	6/1/2041	1,727,725.00	867,093.61
6/1/2029	1,403,225.00	1,093,885.50	12/1/2041	220,975.00	108,884.39
12/1/2029	562,725.00	430,697.03	6/1/2042	1,760,975.00	851,935.65
6/1/2030	1,422,725.00	1,069,123.19	12/1/2042	182,475.00	86,673.85
12/1/2030	541,225.00	399,315.12	6/1/2043	1,802,475.00	840,591.68
6/1/2031	1,441,225.00	1,044,000.75	12/1/2043	150,075.00	68,715.59
12/1/2031	518,725.00	368,924.35	6/1/2044	1,835,075.00	824,958.10
6/1/2032	1,463,725.00	1,022,093.78	12/1/2044	116,375.00	51,365.19
12/1/2032	495,100.00	339,433.97	6/1/2045	1,866,375.00	808,796.41
6/1/2033	1,490,100.00	1,003,018.41	12/1/2045	81,375.00	34,622.82
12/1/2033	470,225.00	310,763.73	6/1/2046	1,901,375.00	794,273.95
6/1/2034	1,515,225.00	983,179.59	12/1/2046	47,250.00	19,379.18
			6/1/2047	1,937,250.00	780,100.30
					<b>\$49,342,156.53</b>
					<b>\$32,003,792.11</b>

## Step #3: Calculate All-In T.I.C.

Total PV:	<b>\$32,003,792.11</b>
All-In T.I.C.:	<b>3.703682%</b>

# Calculation of N.I.C.

The N.I.C. is the blended cost of borrowing that factors in the average interest rate weighted for the time to maturity and does NOT factor in the time value of money.

## Step #1: Calculate Numerator

Total Interest Payments:	\$20,367,156.53
+ Underwriter's Discount:	123,991.85
- Premium:	<u>3,619,643.90</u>
Total:	\$16,871,504.48

## Step #2: Calculate the Denominator

<u>Maturity</u>	<u>Principal</u>	<u>Yrs. From Dated Date</u>	<u>Bond Years</u>
6/1/2023	\$640,000	0.79	503,111
6/1/2024	640,000	1.79	1,143,111
6/1/2025	675,000	2.79	1,880,625
6/1/2026	705,000	3.79	2,669,208
6/1/2027	740,000	4.79	3,541,722
6/1/2028	780,000	5.79	4,513,167
6/1/2029	820,000	6.79	5,564,611
6/1/2030	860,000	7.79	6,696,056
6/1/2031	900,000	8.79	7,907,500
6/1/2032	945,000	9.79	9,247,875
6/1/2033	995,000	10.79	10,732,181
6/1/2034	1,045,000	11.79	12,316,486
6/1/2035	1,095,000	12.79	14,000,792
6/1/2036	1,150,000	13.79	15,854,028
6/1/2037	1,210,000	14.79	17,891,194
6/1/2038	1,270,000	15.79	20,048,361
6/1/2039	1,330,000	16.79	22,325,528
6/1/2040	1,400,000	17.79	24,900,556
6/1/2041	1,470,000	18.79	27,615,583
6/1/2042	1,540,000	19.79	30,470,611
6/1/2043	1,620,000	20.79	33,673,500
6/1/2044	1,685,000	21.79	36,709,597
6/1/2045	1,750,000	22.79	39,875,694
6/1/2046	1,820,000	23.79	43,290,722
6/1/2047	<u>1,890,000</u>	24.79	<u>46,845,750</u>
Total:	\$28,975,000		440,217,569

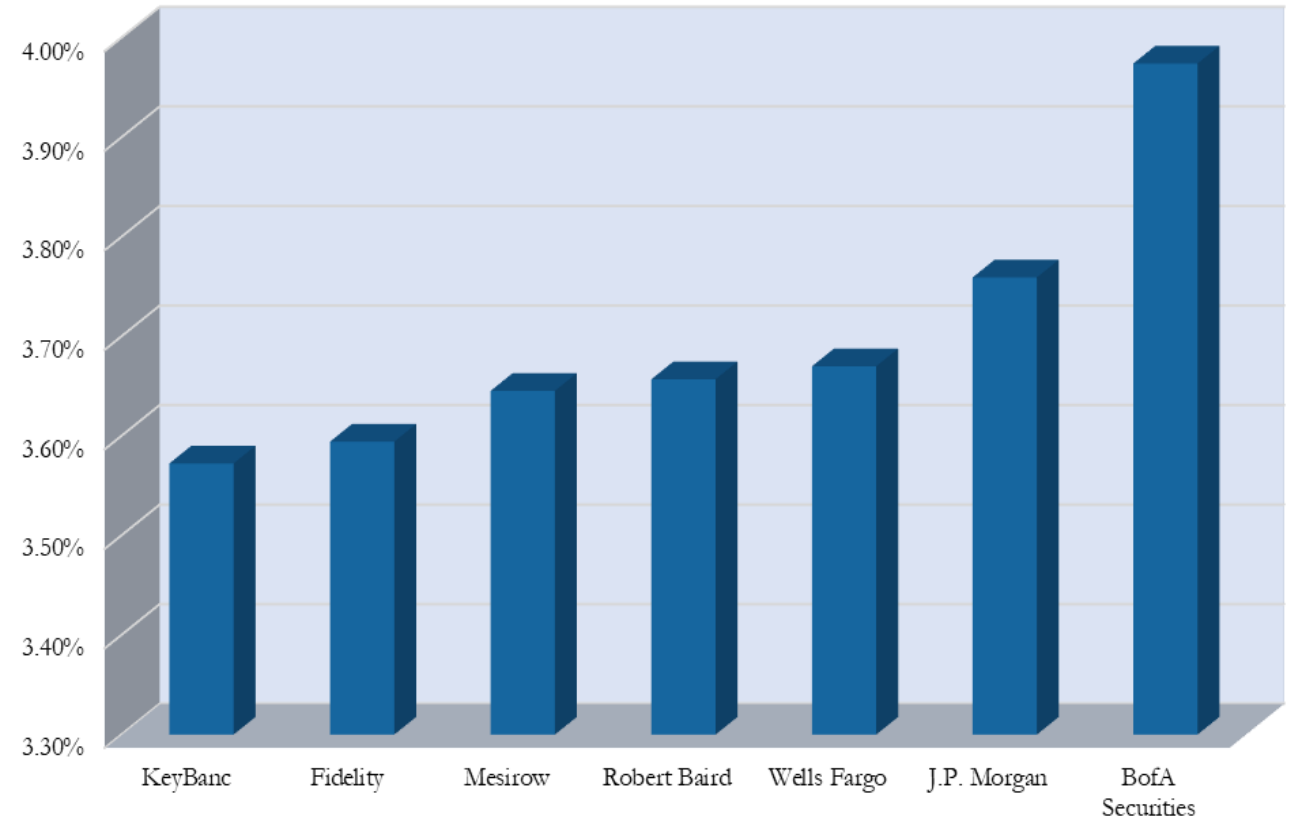
## Step #3: Calculate N.I.C.

Numerator:	\$16,871,504.48
Denominator (Bond Years):	440,217,569.44
N.I.C.:	<b>3.832538%</b>

# Pricing Via Competitive Sale

This transaction was priced via competitive sale, with T.I.C. as the basis for award.

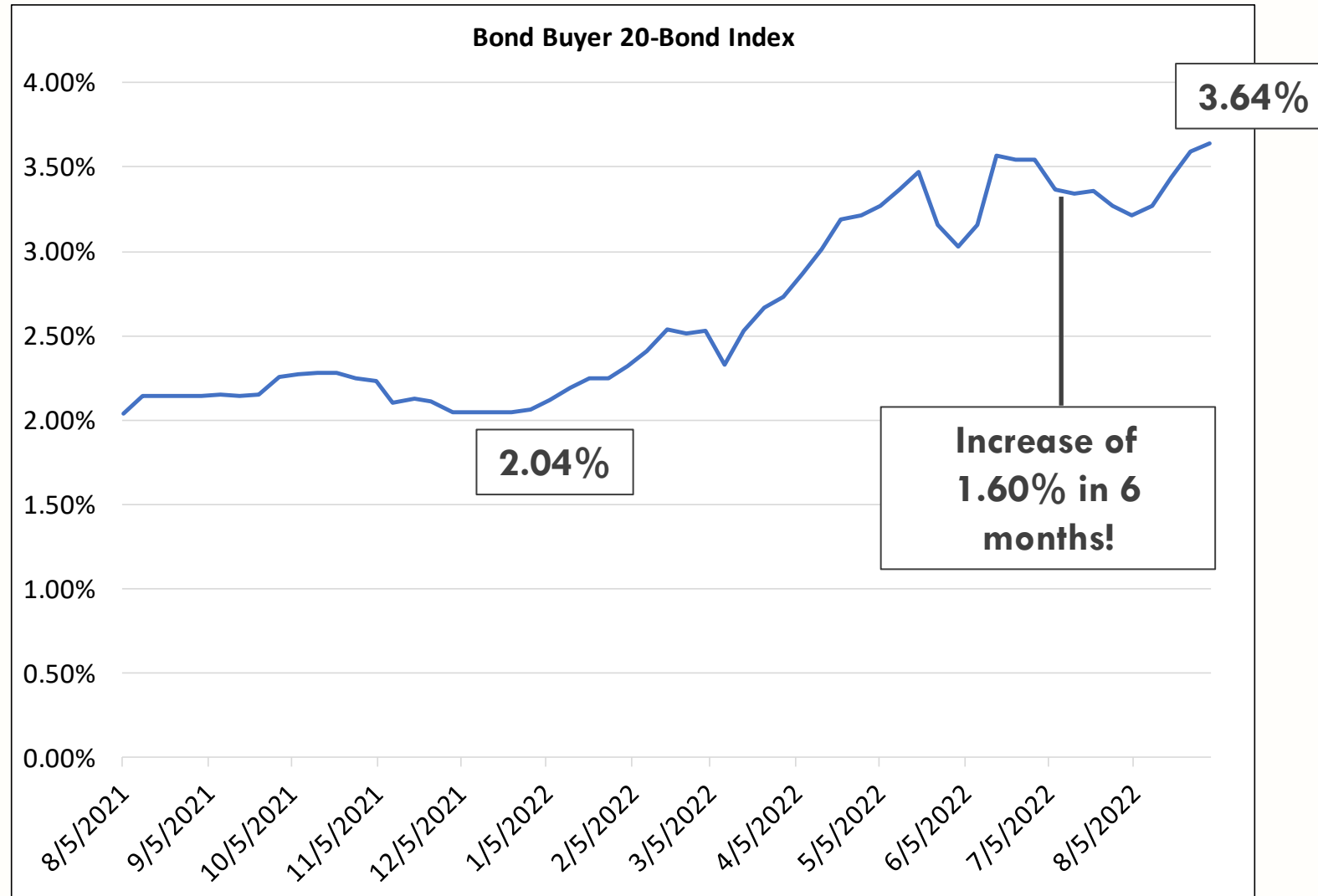
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<input type="checkbox"/>	<a href="#">Fidelity Capital Markets</a>	3.594273
<input type="checkbox"/>	<a href="#">Mesirow Financial, Inc.</a>	3.645351
<input type="checkbox"/>	<a href="#">Robert W. Baird &amp; Co., Inc.</a>	3.656948
<input type="checkbox"/>	<a href="#">Wells Fargo Bank, National Association</a>	3.669999
<input type="checkbox"/>	<a href="#">J.P. Morgan Securities LLC</a>	3.759251
<input type="checkbox"/>	<a href="#">BofA Securities</a>	3.974176





## Section 3: The Impact of Recent Market Movements

# Shift in Bond Market in 2022

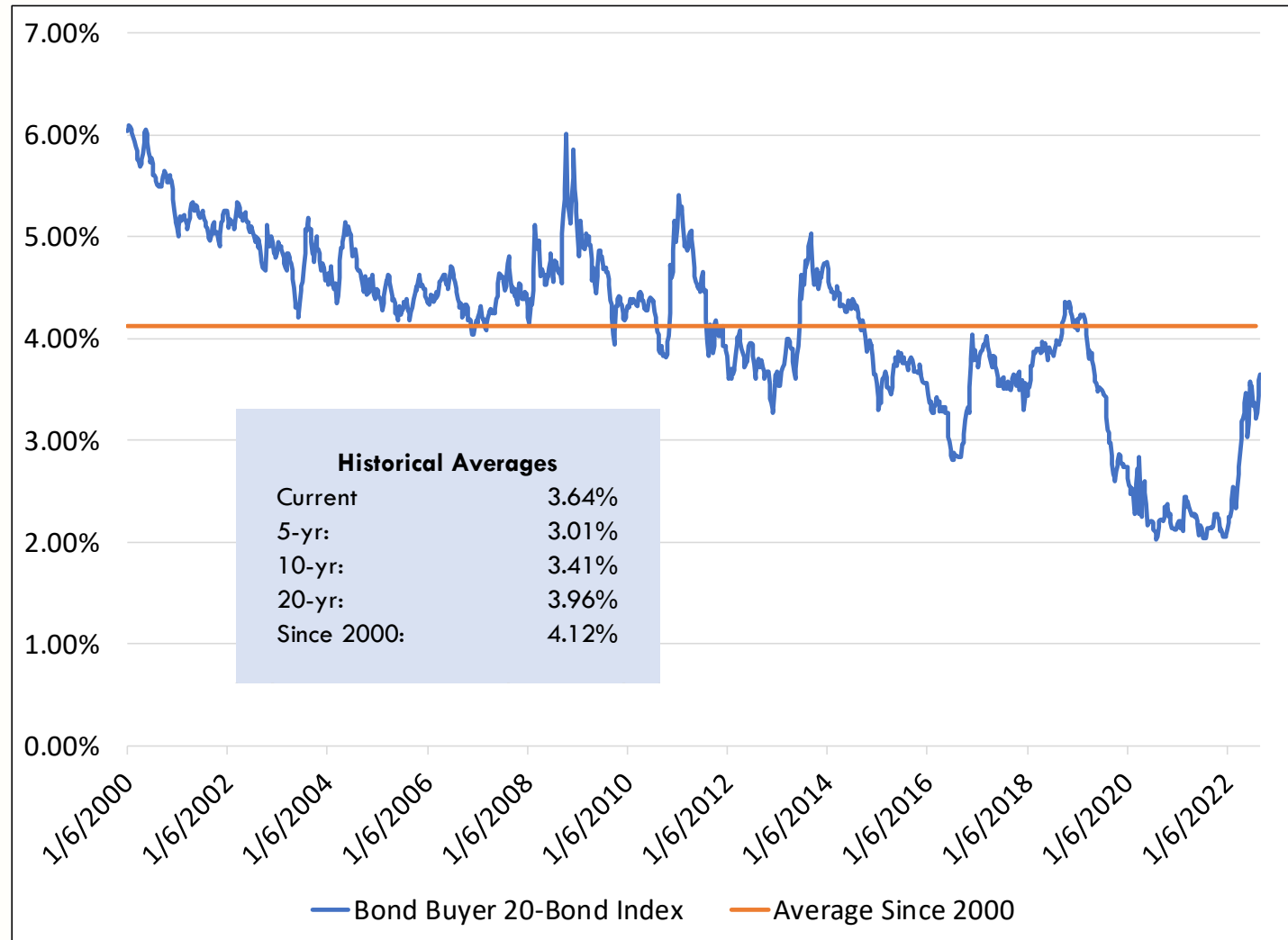


Source: *The Bond Buyer*.

\*General obligation bonds maturing in 20 years are used in compiling the indexes. The 20-bond index has an average rating equivalent to Moody's Aa2 and S&P's AA.



# Rates from a Historical Perspective

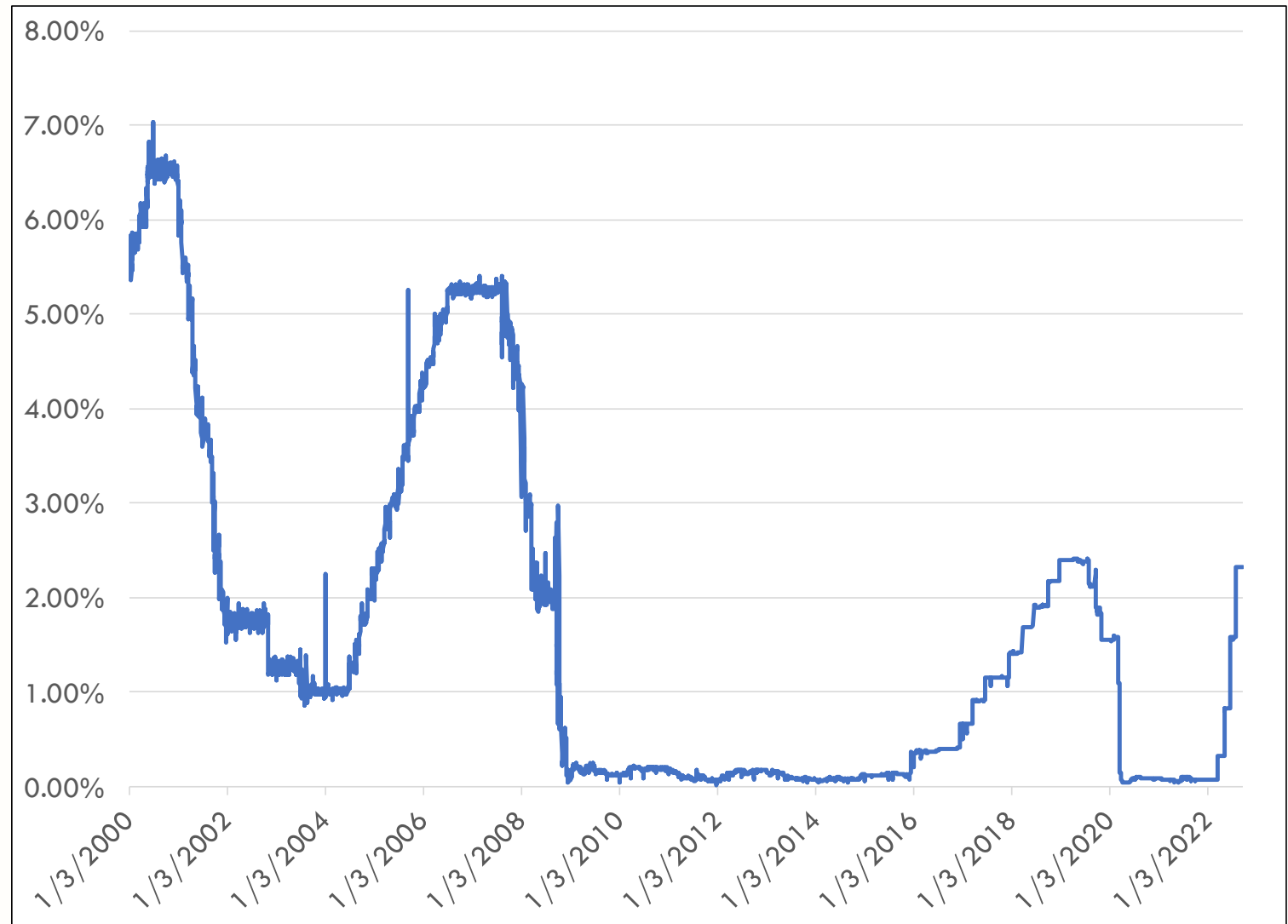


Source: *The Bond Buyer*.

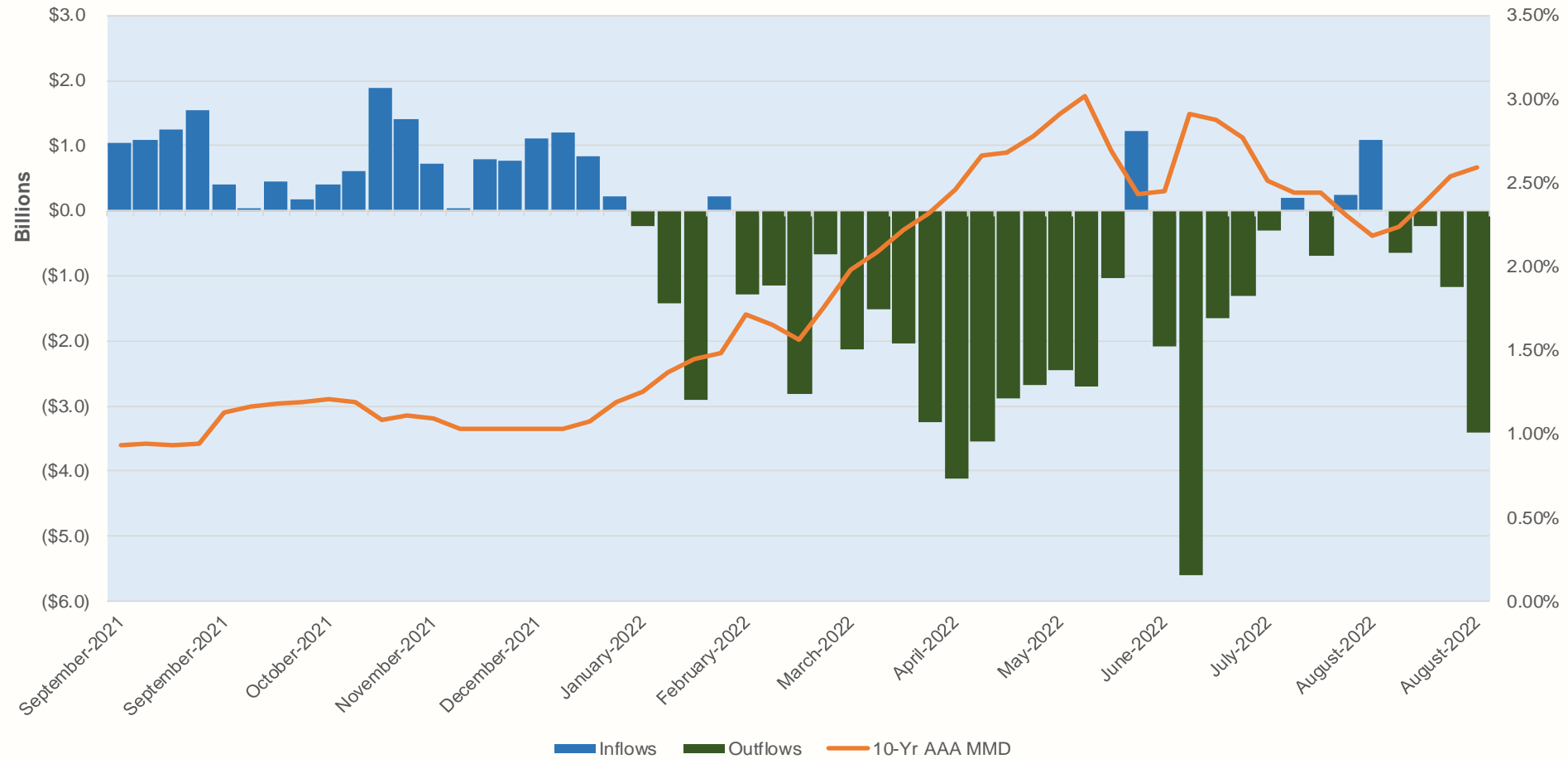
\*General obligation bonds maturing in 20 years are used in compiling the indexes. The 20-bond index has an average rating equivalent to Moody's Aa2 and S&P's AA.

# Proactive FOMC to Target Inflation

- The FOMC has increased the target for the federal funds rate in 2022.
- There are two more FOMC meetings in 2022
  - November 1-2
  - December 13-14



# Municipal Bond Fund Flows Impact Market Demand



Source: Refinitiv / TM3, Lipper.



# Implications of Rising Rate Environment

## Borrower's Perspective:

- Higher cost of funds for new money projects.
- Lower “debt capacity”.
- Less opportunity to generate savings from refundings.

## Investor's Perspective:

- Declining value in bond portfolios.
- Higher yielding new investments in bond portfolios.



## Section 4: Frequently Asked Questions



## FAQ #1

Question:

- Why are certain bonds priced a premium, par, or discount?

Answer:

- Largely depends on investor preference and market conditions:
- Institutional investors, who buy and actively trade bonds, are typically purchasers of premium tax-exempt bonds.
- Individual investors, who typically buy and hold to maturity, are typically purchasers of par or discount tax-exempt bonds.



## FAQ #2

### Question:

- Why are new issue tax-exempt bonds typically priced with a premium structure?

### Answer:

- The premium pricing structure is so pervasive, viewed as a market convention.
- Viewed as “defensive” couponing structure; dollar price is less sensitive to market movement. (example on next page)
- Premium bond pricing helps prevent the triggering of a tax event due to the *de minimis* rule.
  - A discount bond may trigger a taxable event (either as capital gains or ordinary income) and investors who purchase tax-exempt bonds generally want to avoid such situations.

# FAQ #2 - Example of Defensive Couponing

## Example A: Premium Bond

Maturity	Par	Coupon	Yield	Dollar Price
6/1/2042	1,540,000	5.00%	3.21%	114.926
Yield Increase of 50 bps:			3.71%	110.500
Reduction in Dollar Price:				-4.426
Reduction in Dollar Price as %:				-3.9%

## Example B: Discount Bond

Maturity	Par	Coupon	Yield	Dollar Price
6/1/2042	1,540,000	3.00%	3.21%	96.938
Yield Increase of 50 bps:			3.71%	90.106
Reduction in Dollar Price:				-6.832
Reduction in Dollar Price as %:				-7.0%





## FAQ #3

### Question:

- Should I dictate premium couponing structure on my new money transactions in order to preserve the refundability in the future?

### Answer:

- Depends on multiple factors:
  - Future interest rate environment, which no one can predict.
  - Remaining term and par amount of financing at call date.
  - Pricing benefit between premium vs. par/discount bonds.
  - Size of transaction or frequency of issuance; are you likely to execute a refunding?

# FAQ #3 – Sample Analysis

- Analysis can help to dimension the trade-offs between couponing structures.

## Example Analysis Comparing “Low Coupon” Structure vs. 4.00% Coupon Structure

Low Coupon Vs. 4% Scale Scenarios	
4/1/2021-4/1/2030 Debt Service Comparison	
Low Coupon Scenario DS:	17,225,634.03
4% Scale Scenario DS:	17,793,088.89
Difference:	-567,454.86
<b>Low Coupon Scenario Saves:</b>	<b>567,454.86</b>

Low Coupon Vs. 4% Scale Scenarios			
Assuming 10-Year Par Call is Not Used			
PV of Low Coupon Scenario's DS:	38,018,678.48	Gross DS of Low Coupon Scenario:	59,208,409.03
(Less) PV of 4% Scale Scenario's DS:	38,613,433.41	(Less) Gross DS of 4% Scale Scenario:	59,847,288.89
Difference in PV:	-594,754.93	Difference in Gross DS:	-638,879.86
<b>Low Coupon Scenario Saves:</b>	<b>594,754.93</b>	<b>Low Coupon Scenario Saves:</b>	<b>638,879.86</b>

Low Coupon Vs. 4% Scale Scenarios			
Assuming 10-Year Par Call is Used			
PV of Low Coupon Scenario's DS:	35,549,506.03	Gross DS of Low Coupon Scenario:	55,111,309.59
(Less) PV of 4% Scale Scenario's DS:	35,187,187.17	(Less) Gross DS of 4% Scale Scenario:	54,014,363.33
Difference in PV:	362,318.86	Difference in Gross DS:	1,096,946.26
<b>4% Scale Scenario Saves:</b>	<b>362,318.86</b>	<b>4% Scale Scenario Saves:</b>	<b>1,096,946.26</b>

## Section 5: Audience Q&A



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# Appendix: Detailed Bond Price Calculation

# Appendix: Detailed Bond Price

- The dollar price of a bond is the present value of the future cashflows at the market yield
- Coupon, yield, and time are the only factors in price
- There are several methods to derive bond price, including:

**Bond Dollar Price**

$$= \frac{100}{\left(1 + \frac{\text{Yield}}{2}\right)^{\# \text{ of semi-annual periods}}} + \frac{100 \times \text{Coupon} \times \left[1 - \left(1 + \frac{\text{Yield}}{2}\right)^{-\# \text{ of semi-annual periods}}\right]}{\left(\frac{\text{Yield}}{2}\right)}$$

Present value of the principal at maturity
Present value of the interest payments over time

# Bond Price Formula Example

- What is the price of a municipal bond assuming:
  - 10 Year Maturity
  - 5.00% Coupon
  - 4.00% Yield

$$\text{Bond Price} = \frac{100}{\left(1 + \frac{\text{Yield}}{2}\right)^{\# \text{ of semi-annual periods}}} + 100 \times \frac{\text{Coupon}}{2} \times \frac{1 - \left(1 + \frac{\text{Yield}}{2}\right)^{-\# \text{ of semi-annual periods}}}{\left(\frac{\text{Yield}}{2}\right)}$$

# Bond Price Formula Example

- What is the price of a municipal bond assuming:
  - 10 Year Maturity (20 semi-annual periods)
  - 5.00% Coupon
  - 4.00% Yield

**Bond Price**

$$= \frac{100}{\left(1 + \frac{\text{Yield}}{2}\right)^{20}} + 100 \times \frac{\text{Coupon}}{2} \times \frac{1 - \left(1 + \frac{\text{Yield}}{2}\right)^{-20}}{\left(\frac{\text{Yield}}{2}\right)}$$

# Bond Price Formula Example

- What is the price of a municipal bond assuming:
  - 10 Year Maturity (20 semi-annual periods)
  - **5.00% Coupon**
  - 4.00% Yield

**Bond Price** =  $\frac{100}{\left(1 + \frac{\text{Yield}}{2}\right)^{20}} + 100 \times .025 \times \frac{1 - \left(1 + \frac{\text{Yield}}{2}\right)^{-20}}{\left(\frac{\text{Yield}}{2}\right)}$



# Bond Price Formula Example

- What is the price of a municipal bond assuming:
  - 10 Year Maturity (20 semi-annual periods)
  - 5.00% Coupon
  - 4.00% Yield

**Bond Price**

$$= \frac{100}{\left(1 + \frac{.04}{2}\right)^{20}} + 100 \times .025 \times \frac{1 - \left(1 + \frac{.04}{2}\right)^{-20}}{\left(\frac{.04}{2}\right)}$$

The diagram shows three arrows originating from the text '4.00% Yield' in the list above. One arrow points to the '.04' in the denominator of the first fraction. A second arrow points to the '.04' in the numerator of the second fraction. A third arrow points to the '.04' in the denominator of the second fraction.

# Bond Price Formula Example

- Municipal bond convention for pricing is truncation at the 3<sup>rd</sup> decimal
- No rounding!

$$\text{Bond Price} = 108.1757\text{X}66$$

# MSRB Rule G-42: Disclosure of Conflicts of Interest & Legal or Disciplinary Events

Pursuant to Municipal Securities Rulemaking Board (“MSRB”) Rule G-42, on Duties of Non-Solicitor Municipal Advisors, Municipal Advisors are required to make certain written disclosures to clients which include, amongst other things, Conflicts of Interest and any Legal or Disciplinary events of KNN Public Finance, LLC (“KNN Public Finance”) and its associated persons.

## Conflicts of Interest

**Other Municipal Advisor Relationships.** KNN serves a wide variety of other clients that may from time to time have interests that could have a direct or indirect impact on the interests of another KNN client. For example, KNN serves as municipal advisor to other municipal advisory clients and, in such cases, owes a regulatory duty to such other clients just as it will to your entity, if hired. These other clients may, from time to time and depending on the specific circumstances, have competing interests. In acting in the interests of its various clients, KNN could potentially face a conflict of interest arising from these competing client interests. KNN fulfills its regulatory duty and mitigates such conflicts through dealing honestly and with the utmost good faith with its clients.

**Compensation.** KNN Public Finance represents that in connection with the issuance of municipal securities, KNN Public Finance may receive compensation from an Issuer or Obligated Person for services rendered, which compensation is contingent upon the successful closing of a transaction and/or is based on the size of a transaction. Consistent with the requirements of MSRB Rule G-42, KNN Public Finance hereby discloses that such contingent and/or transactional compensation may present a potential conflict of interest regarding KNN Public Finance’s ability to provide unbiased advice to enter into such transaction. This conflict of interest will not impair KNN Public Finance’s ability to render unbiased and competent advice or to fulfill its fiduciary duty to the Issuer.

If KNN Public Finance becomes aware of any additional potential or actual conflict of interest after this disclosure, KNN Public Finance will disclose the detailed information in writing to the Issuer in a timely manner.

## Legal or Disciplinary Events

KNN Public Finance, LLC, has never been subject to any legal, disciplinary or regulatory actions nor was it ever subject to any legal, disciplinary or regulatory actions previously, when it was a division of Zions First National Bank or Zions Public Finance, Inc.

A regulatory action disclosure has been made on Form MA-I for one of KNN Public Finance municipal advisory personnel relating to a 1998 U.S. Securities and Exchange Commission (“SEC”) order that was filed while the municipal advisor was employed with a prior firm, (not KNN Public Finance). The details of which are available in Item 9; C(1), C(2), C(4), C(5) and the corresponding regulatory action DRP section on Form MA and Item 6C; (1), (2), (4), (5) and the corresponding regulatory action DRP section on Form MA-I. Issuers may electronically access KNN Public Finance’s most recent Form MA and each most recent Form MA-I filed with the Commission at the following website: [www.sec.gov/edgar/searchedgar/companysearch.html](http://www.sec.gov/edgar/searchedgar/companysearch.html).

The SEC permits certain items of information required on Form MA and Form MA-I to be provided by reference to such required information already filed on a regulatory system (e.g., FINRA CRD). The above noted regulatory action has been referenced on both Form MA and MA-I due to the information already filed on FINRA’s CRD system and is publicly accessible through BrokerCheck at <http://brokercheck.finra.org>. For purposes of accessing such BrokerCheck information, the Municipal Advisor’s CRD number is 4457537.

There has been no change to any legal or disciplinary event that has been disclosed on KNN Public Finance’s original SEC registration Form MA filed on February 8, 2016 or Form MA-I’s filed on January 22, 2016.