CDIAC

CALIFORNIA
DEBT AND
INVESTMENT
ADVISORY
COMMISSION

ECONOMICS AND STRUCTURES

INTERMEDIATE BOND MATH (PART 2)

PRESENTED BY LOUIS CHOI

PUBLIC RESOURCES ADVISORY GROUP

AN INDEPENDENT REGISTERED MUNICIPAL ADVISOR (IRMA)

August 20, 2014 2:00 PM – 3:30 PM

Topics

- How Do Refundings Work?
- Economics of Callable Bonds
- Non-Callable Bonds
- CABs and Convertible CABs
- Bonus: Valuing Call Options

How Do Refundings Work?

Economics and structures

intermediate bond math (Part 2)

Refinancing for Savings

Starting with a Loan…

Assumptions –

- $50,000,000 outstanding balance
- Repaid in 5 years
- Original interest rate of 5.00%; new interest rate of 4.00%
- Refinancing fees of $500,000 for new loan

	Original Loan			New Loan			
		5.00%	Debt		3.00%	Debt	
Date	Principal	Interest	Service	Principal	Interest	Service	Savings
5/1/2014							
5/1/2015	9,048,740	2,500,000	11,548,740	9,511,906	1,515,000	11,026,906	521,834
5/1/2016	9,501,177	2,047,563	11,548,740	9,797,263	1,229,643	11,026,906	521,834
5/1/2017	9,976,236	1,572,504	11,548,740	10,091,181	935,725	11,026,906	521,834
5/1/2018	10,475,048	1,073,692	11,548,740	10,393,916	632,990	11,026,906	521,834
5/1/2019	10,998,800	549,940	11,548,740	10,705,734	321,172	11,026,906	521,834
Total	50,000,000	7,743,700	57,743,700	50,500,000	4,634,529	55,134,529	2,609,170

Sources of Funds	
New Loan Principal	50,500,000
Total Sources of Funds	50,500,000
Uses of Funds	
Original Loan Principal	50,000,000
New Loan Fees	500,000
Total Uses of Funds	50,500,000

Result:

- Higher new principal amount to cover closing costs
- Difference in interest produce savings

…Converting to Bonds…

	Original Loan				New Loan						
Date	Principal	Coupon	Interest	Debt Service	Principal	Coupon	Interest	Debt Service	Yield	Proceeds	Savings
5/1/2014											
5/1/2015	9,120,000	4.50%	2,381,850	11,501,850	9,625,000	2.00%	1,264,675	10,889,675	2.00%	9,625,000	612,175
5/1/2016	9,530,000	4.50%	1,971,450	11,501,450	9,820,000	2.25%	1,072,175	10,892,175	2.25%	9,820,000	609,275
5/1/2017	9,960,000	4.75%	1,542,600	11,502,600	10,040,000	2.50%	851,225	10,891,225	2.50%	10,040,000	611,375
5/1/2018	10,435,000	5.00%	1,069,500	11,504,500	10,290,000	2.75%	600,225	10,890,225	2.75%	10,290,000	614,275
5/1/2019	10,955,000	5.00%	547,750	11,502,750	10,575,000	3.00%	317,250	10,892,250	3.00%	10,575,000	610,500
Total	50,000,000		7,513,150	57,513,150	50,350,000		4,105,550	54,455,550		50,350,000	3,057,600

Sources of Funds	
Principal	50,350,000
Net OIP / (OID)	0
Total Sources of Funds	50,350,000
Uses of Funds	
Original Principal Repayment	50,000,000
Costs of Issuance	225,000
Underwriter's Discount	121,200
Contingency	3,800
Total Uses of Funds	50,350,000

Steps:

- Round principal amounts by denomination
- Introduce multiple interest rates (i.e., coupons)
- Calculate proceeds, costs of issuance and underwriter's discount
- Adjust principal of each maturity to target proceeds

[Hint: See slides 21 to 27 of Intermediate Bond Math 1]

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...Adjusting Coupons...

	Original Loan				New Loan						
Date	Principal	Coupon	Interest	Debt Service	Principal	Coupon	Interest	Debt Service	Yield	Proceeds	Savings
5/1/2014											
5/1/2015	9,120,000	4.50%	2,381,850	11,501,850	8,470,000	5.00%	2,417,788	10,887,788	2.00%	8,720,289	614,063
5/1/2016	9,530,000	4.50%	1,971,450	11,501,450	8,895,000	5.00%	1,994,288	10,889,288	2.25%	9,370,705	612,163
5/1/2017	9,960,000	4.75%	1,542,600	11,502,600	9,340,000	5.25%	1,549,538	10,889,538	2.50%	10,077,860	613,063
5/1/2018	10,435,000	5.00%	1,069,500	11,504,500	9,830,000	5.25%	1,059,188	10,889,188	2.75%	10,754,806	615,313
5/1/2019	10,955,000	5.00%	547,750	11,502,750	10,345,000	5.25%	543,113	10,888,113	3.00%	11,418,190	614,638
Total	50,000,000		7,513,150	57,513,150	46,880,000		7,563,913	54,443,913		50,341,850	3,069,238

Sources of Funds	
Principal	46,880,000
Net OIP / (OID)	3,461,850
Total Sources of Funds	50,341,850
Uses of Funds	
Original Principal Repayment	50,000,000
Costs of Issuance	225,000
Underwriter's Discount	114,260
Contingency	2,590
Total Uses of Funds	50,341,850

Observations

- Yields, rather than coupons, are the primary driver of savings generated in a refunding
- Increasing coupons raise prices, allowing for the issuance of less principal, reducing refunding debt service and preserving savings

...Calculating Net Present Value Savings...

	Original Loan				New Loan							Present
				Debt				Debt				Value of
Date	Principal	Coupon	Interest	Service	Principal	Coupon	Interest	Service	Yield	Proceeds	Savings	Savings
5/1/2014												
5/1/2015	9,120,000	4.50%	2,381,850	11,501,850	7,255,000	5.00%	2,071,738	9,326,738	2.00%	7,469,385	2,175,113	2,099,945
5/1/2016	9,530,000	4.50%	1,971,450	11,501,450	7,620,000	5.00%	1,708,988	9,328,988	2.25%	8,027,518	2,172,463	2,021,848
5/1/2017	9,960,000	4.75%	1,542,600	11,502,600	8,005,000	5.25%	1,327,988	9,332,988	2.50%	8,637,395	2,169,613	1,946,470
5/1/2018	10,435,000	5.00%	1,069,500	11,504,500	8,425,000	5.25%	907,725	9,332,725	2.75%	9,217,624	2,171,775	1,878,190
5/1/2019	10,955,000	5.00%	547,750	11,502,750	8,865,000	5.25%	465,413	9,330,413	3.00%	9,784,655	2,172,338	1,810,767
Total	50,000,000		7,513,150	57,513,150	40,170,000		6,481,850	46,651,850		43,136,577	10,861,300	9,757,220

Sources of Funds	
Principal	40,170,000
Net OIP / (OID)	2,966,577
Original Funds on Hand	11,504,500
Total Sources of Funds	54,641,077
Uses of Funds	
Original Principal Repayment	50,000,000
Reserve Fund	4,313,658
Costs of Issuance	225,000
Underwriter's Discount	100,840
Contingency	1,579
Total Uses of Funds	54,641,077

Present Value of Cash Flow Savings	9,757,220
less:	
Original Funds on Hand Used	-11,504,500
plus:	
New Reserve Fund	4,313,658
Contingency	1,579
Net Present Value Savings	2,567,957
NPV Savings as % of Original Principal	5.14%

Savings Adjustments

- Any funds contributed into or generated by the refunding must be included
- Cash flow savings must be translated to delivery-date dollars using "time value of money" approach*

* Discounting follows municipal bond conventions using 30/360-day count and semi-annual compounding and is typically done at the arbitrage yield *[Hint: see slide 26 of Intermediate Bond Math 1]*

Advance Refunding

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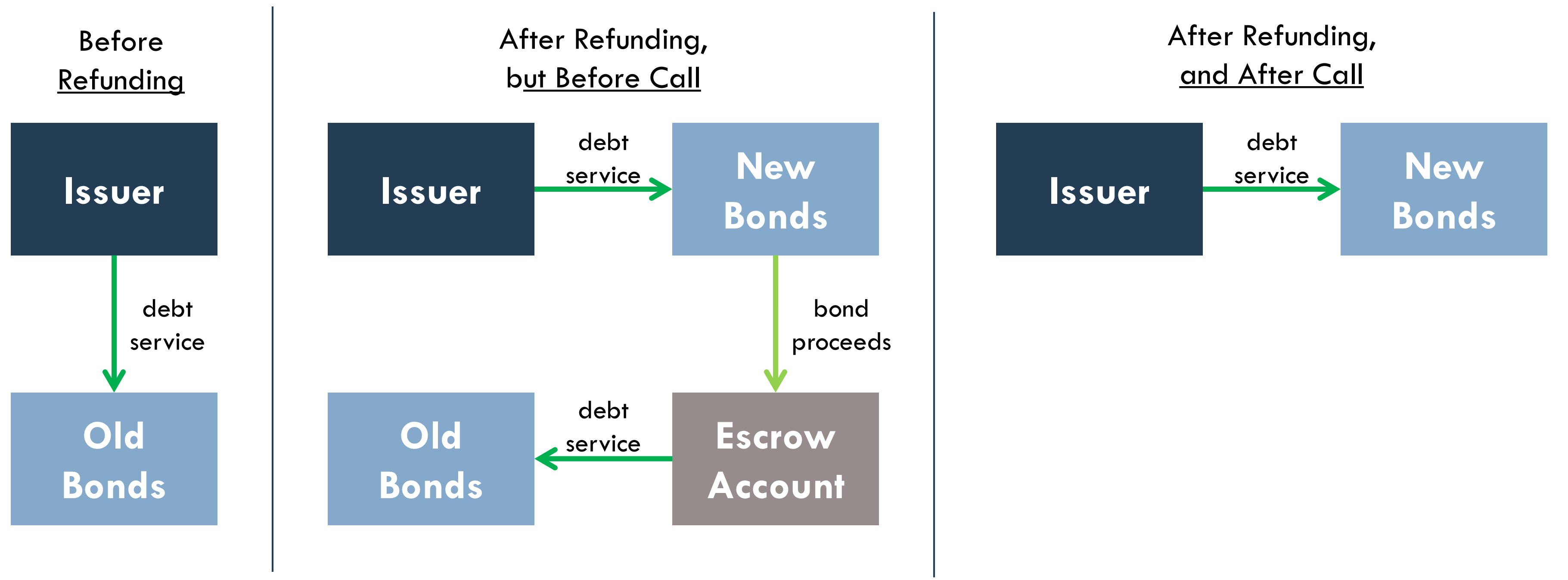
What is it?

- A refunding in which the new bonds are delivered more than 90 days in advance of the call date of the old (refunded) bonds
- An escrow needs to be established to fund principal and interest due on the old bonds
- Note: There are special IRS rules related to advance refundings



…and Calculating Escrow Requirements and Escrow Cost

	Original Loan				New Loan							Present
				Debt				Debt				Value of
Date	Principal	Coupon	Interest	Service	Principal	Coupon	Interest	Service	Yield	Proceeds	Savings	Savings
5/1/2014												
5/1/2015	9,120,000	4.50%	2,381,850	11,501,850	7,255,000	5.00%	2,071,738	9,326,738	2.00%	7,469,385	2,175,113	2,099,945
5/1/2016	9,530,000	4.50%	1,971,450	11,501,450	7,620,000	5.00%	1,708,988	9,328,988	2.25%	8,027,518	2,172,463	2,021,848
5/1/2017	9,960,000	4.75%	1,542,600	11,502,600	8,005,000	5.25%	1,327,988	9,332,988	2.50%	8,637,395	2,169,613	1,946,470
5/1/2018	10,435,000	5.00%	1,069,500	11,504,500	8,425,000	5.25%	907,725	9,332,725	2.75%	9,217,624	2,171,775	1,878,190
5/1/2019	10,955,000	5.00%	547,750	11,502,750	8,865,000	5.25%	465,413	9,330,413	3.00%	9,784,655	2,172,338	1,810,767
Total	50,000,000		7,513,150	57,513,150	40,170,000		6,481,850	46,651,850		43,136,577	10,861,300	9,757,220

	Redeemed			Escrow	Escrow Securities					
Date	Principal		Interest	Requirement	Principal	Coupon	Interest	Cash Flow	Yield	Cost
5/1/2014										
11/1/2014			1,190,925	1,190,925	515,331	2.40%	675,594	1,190,925	2.40%	515,331
5/1/2015	50,000,000		1,190,925	51,190,925	50,521,515	2.65%	669,410	51,190,925	2.65%	50,521,515
Total	50,000,000		2,381,850	52,381,850	51,036,846		1,345,004	52,381,850		51,036,846

Hint: Approach to modeling escrow requirements and escrow cost is very similar to how other debt service schedules are calculated

Sources of Funds	
Principal	40,170,000
Net OIP / (OID)	2,966,577
Original Funds on Hand	11,504,500
Total Sources of Funds	54,641,077
Uses of Funds	
Original Principal Repayment	51,036,846
Reserve Fund	4,313,658
Costs of Issuance	225,000
Underwriter's Discount	100,840
Contingency	-1,035,267
Total Uses of Funds	54,641,077

Present Value of Cash Flow Savings	9,757,220
less:	
Original Funds on Hand Used	-11,504,500
plus:	
New Reserve Fund	4,313,658
Contingency	-1,035,267
Net Present Value Savings	1,531,111
NPV Savings as % of Original Principal	3.06%

Observations:

- Escrow inefficiency reduces savings
- Including non-callable bonds also reduce savings

Estimating Refunding Savings

- Understanding refunding cash flows and that TVM is the basis for pricing bonds, it is possible to estimate savings by combining two price functions
- The proof is as follows:

$$
\begin{aligned}
NPV(Savings) &= PV_{new}(DS_{old}) - PV_{new}(DS_{new}) \\
&= PV_{new}(DS_{old,\,per\$100}) \times P_{old} - PV_{new}(DS_{new,\,per\$100}) \times P_{new} \\
&= PV_{new}(DS_{old,\,per\$100}) \times P_{old} - PV_{new}(DS_{new,\,per\$100}) \times \frac{P_{old} \times Cost_{esc}}{(1 - COI_{new})} \\
&= \left(PV_{new}(DS_{old,\,per\$100}) - PV_{new}(DS_{new,\,per\$100}) \times \frac{Cost_{esc}}{(1 - COI_{new})} \right) \times P_{old} \\
&= \left(PV_{new}(DS_{old,\,per\$100}) - PV_{new}(DS_{new,\,per\$100}) \times \frac{PV_{esc}(CF_{esc})}{(1 - COI_{new})} \right) \times P_{old} \\
&= \left(PRICE(Bond_{old}, Rate_{new}) - 100\% \times \frac{PRICE(Bond_{old,\,tocall}, Rate_{esc})}{(1 - COI_{new})} \right) \times P_{old} \\
&= \left(PRICE(Bond_{old}, Rate_{new}) - \frac{PRICE(Bond_{old,\,tocall}, Rate_{esc})}{(1 - COI_{new})} \right) \times P_{old}
\end{aligned}
$$

Using Excel to Estimate Refunding Savings

	A	B
1	Delivery	5/14/2014
2	Maturity	5/1/2020
3	Old Coupon	5.00%
4	New Rate	2.65%
5	Call Date	5/1/2015
6	Escrow Yield	0.25%
7	Call Price	100
8	COI	0.8%

NPV Savings % = (
PRICE(B1,B2,B3,B4,100,2) –
PRICE(B1,B5,B3,B6,B7,2)/
(1 – B9)) / 100

Tip:
New rate is yield-to-maturity of refunding bond.

Economics of Callable Bonds

Economics and structures

intermediate bond math (Part 2)

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What Do Yields Really Mean?

Amortizing Premiums and Discounts:

- Over time, prices drift towards the par value of the bonds (which is 100% of principal) and the premium or discount is said to "amortize"
- For an investor, the earnings is equal to the interest received plus the change in the value of the bond

$$E_n = P \times \underbrace{(PR_n - PR_{n-1} + C)}_{\text{Amortization of premium or discount}}$$

- "E_n" = Earnings in year n
- "P" = Principal held
- "P_n" = Price in year n
- "C" = Coupon

Example:

Principal: $100,000

Coupon: 4.00%

Yield: 3.50%

Date1: 5/1/2014

Date2: 5/1/2015

Prices to maturity on different dates based on the same yield of 3.50%

$$E = \$100{,}000 \times (106.897\% - 107.149\% + 4.00\%)$$

= $3,748.01 or 3.50% of $107,149 invested

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What Do Yields Really Mean?

Example:
Maturity: 5/1/2034
Coupon: 4.000%
Yield: $\boxed{3.500\%}$

same

Date	Value Based on Maturity	Change in Value	Coupon Received	Total "Interest" Received	Total "Interest" / Prior Value
5/1/2014	107.149%				
5/1/2015	106.897%	(0.252%)	4.000%	3.748%	3.500%
5/1/2016	106.636%	(0.261%)	4.000%	3.739%	3.500%
5/1/2017	106.366%	(0.270%)	4.000%	3.730%	3.500%
5/1/2018	106.086%	(0.280%)	4.000%	3.720%	3.500%
5/1/2019	105.796%	(0.290%)	4.000%	3.710%	3.500%
5/1/2020	105.497%	(0.300%)	4.000%	3.700%	3.500%
5/1/2021	105.186%	(0.310%)	4.000%	3.690%	3.500%
5/1/2022	104.865%	(0.321%)	4.000%	3.679%	3.500%
5/1/2023	104.533%	(0.333%)	4.000%	3.667%	3.500%
5/1/2024	104.188%	(0.344%)	4.000%	3.656%	3.500%
5/1/2025	103.832%	(0.357%)	4.000%	3.643%	3.500%
5/1/2026	103.463%	(0.369%)	4.000%	3.631%	3.500%
5/1/2027	103.081%	(0.382%)	4.000%	3.618%	3.500%
5/1/2028	102.685%	(0.396%)	4.000%	3.604%	3.500%
5/1/2029	102.275%	(0.410%)	4.000%	3.590%	3.500%
5/1/2030	101.851%	(0.424%)	4.000%	3.576%	3.500%
5/1/2031	101.412%	(0.439%)	4.000%	3.561%	3.500%
5/1/2032	100.958%	(0.455%)	4.000%	3.545%	3.500%
5/1/2033	100.487%	(0.471%)	4.000%	3.529%	3.500%
5/1/2034	100.000%	(0.487%)	4.000%	3.513%	3.500%

<u>The bottom line:</u>

The yield is constant and equal to the rate of return after accounting for the amortization of premiums and discounts

Yields and Callable Premium Bonds

- Prior to the call date*, stated (or nominal) yield is equal to the rate of return
- After the call date, the rate of return for each period is equal to the coupon

$$E_n = P \times (\underbrace{PR_n - PR_{n-1} + C}_{\text{Equal to the call price for both dates after the call date}})$$

- "E_n" = Earnings in year n
- "P" = Principal held
- "P_n" = Price in year n
- "C" = Coupon

Example:

Principal: $100,000

Coupon: 4.00%

Yield: 3.50%

Date1: 5/1/2024

Date2: 5/1/2025

Prices are based on the call price instead

$$E = \$100{,}000 \times (100.000\% - 100.000\% + 4.00\%)$$

= $4,000.00 or 4.00% of $100,000 invested

* More precisely, the call date to which a bond is priced.

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Yields and Callable Premium Bonds

Example:
- Maturity: 5/1/2034
- Optional Call Date: 5/1/2024
- Optional Call Price: 100%
- Coupon: 4.000%
- Yield: 3.500%

Date	Value Based on Call	Change in Value	Coupon Received	Total "Interest" Received	Total "Interest" / Prior Value	Cumulative Rate of Return
5/1/2014	104.188%					
5/1/2015	103.832%	(0.357%)	4.000%	3.643%	3.497%	3.500%
5/1/2016	103.463%	(0.369%)	4.000%	3.631%	3.497%	3.500%
5/1/2017	103.081%	(0.382%)	4.000%	3.618%	3.497%	3.500%
5/1/2018	102.685%	(0.396%)	4.000%	3.604%	3.497%	3.500%
5/1/2019	102.275%	(0.410%)	4.000%	3.590%	3.497%	3.500%
5/1/2020	101.851%	(0.424%)	4.000%	3.576%	3.496%	3.500%
5/1/2021	101.412%	(0.439%)	4.000%	3.561%	3.496%	3.500%
5/1/2022	100.958%	(0.455%)	4.000%	3.545%	3.496%	3.500%
5/1/2023	100.487%	(0.471%)	4.000%	3.529%	3.496%	3.500%
5/1/2024	100.000%	(0.487%)	4.000%	3.513%	3.496%	3.500%
5/1/2025	100.000%	0.000%	4.000%	4.000%	4.000%	3.537%
5/1/2026	100.000%	0.000%	4.000%	4.000%	4.000%	3.568%
5/1/2027	100.000%	0.000%	4.000%	4.000%	4.000%	3.594%
5/1/2028	100.000%	0.000%	4.000%	4.000%	4.000%	3.616%
5/1/2029	100.000%	0.000%	4.000%	4.000%	4.000%	3.635%
5/1/2030	100.000%	0.000%	4.000%	4.000%	4.000%	3.652%
5/1/2031	100.000%	0.000%	4.000%	4.000%	4.000%	3.667%
5/1/2032	100.000%	0.000%	4.000%	4.000%	4.000%	3.680%
5/1/2033	100.000%	0.000%	4.000%	4.000%	4.000%	3.691%
5/1/2034	100.000%	0.000%	4.000%	4.000%	4.000%	3.702%

Effective Rate per Period

Cumulative Rate

Terminology: *Yield to maturity is the cumulative rate of return for a bond held to maturity*

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Yields and Callable Discount Bonds

Example:
- Maturity: 5/1/2034
- Optional Call Date: 5/1/2024
- Optional Call Price: 100%
- Coupon: 3.500%
- Yield: 3.750%

Date	Value Based on Maturity	Change in Value	Coupon Received	Total "Interest" Received	Total "Interest" / Prior Value
5/1/2014	96.504%				
5/1/2015	96.624%	0.120%	3.500%	3.620%	3.750%
5/1/2016	96.749%	0.125%	3.500%	3.625%	3.750%
5/1/2017	96.878%	0.129%	3.500%	3.629%	3.750%
5/1/2018	97.012%	0.134%	3.500%	3.634%	3.750%
5/1/2019	97.152%	0.139%	3.500%	3.639%	3.750%
5/1/2020	97.296%	0.145%	3.500%	3.645%	3.750%
5/1/2021	97.446%	0.150%	3.500%	3.650%	3.750%
5/1/2022	97.602%	0.156%	3.500%	3.656%	3.750%
5/1/2023	97.764%	0.162%	3.500%	3.662%	3.750%
5/1/2024	100.000%	2.236%	3.500%	5.736%	5.868%

Call Price

Net Gain!

Observation:

If a discount bond is called prior to maturity, including mandatory sinking fund redemptions, the effective cumulative yield for the bondholder would also be above the stated yield

Summary on Callable Bond Economics

Coupon Type	**Par**	**Premium**	**Discount**
Stated (Nominal) Yield	Represents actual yield	Represents yield to *call date*	Represents yield to *maturity*
Yield to Maturity	Represents actual yield	Represents *worst* case scenario	Represents *best* case scenario
Refundings	Neutral	Most likely as savings are highest	Least likely as savings are lowest; incurs "hidden" call premium
Considerations	Should be compared to pricing for "standard" premium coupon bonds	Should be avoided, if refunding in the future is unlikely; could be preferred for bonds whose rates are likely to decline in the future	Discounts can increase cost for refundings in the future; creates "hidden" cost for term bonds

Non-Callable Bonds

Economics and structures

intermediate bond math (Part 2)

Non-Callable Bonds Simplify the Math, But not the Analysis

"Standard" Bond	vs.	Non-Callable Bond
• Generally 5% coupon (premium) • Callable at par after 10 years		• Generally, premium coupon • Non-callable

Advantage →

Effective Yield
Nominal yield = yield-to-maturity

← Advantage

Future Refunding
Potential to realize savings through a future refunding

← Advantage

Tax Law/Arbitrage
Allows certain remediation actions in the event of a change in use

Economic Analysis vs. "Standard" Bonds

	Standard	Non callable
Delivery	10/1/2014	10/1/2014
Maturity	10/1/2034	10/1/2034
Coupon	5.000%	5.000%
Yield	3.660%	3.880%
1st Call Date	10/1/2024	n/a
1st Call Price	100.000	n/a
Price	111.137	115.481
Yield to 10/1/24	3.660%	3.880%
Yield to Maturity	4.173%	3.880%

Generally, premium coupons; investors want to retain "above market" coupons with certainty

In the current market, non-callable bond yields are higher than callable bond nominal yields

Higher price as a result of pricing to maturity

Should be measured against potential refunding savings

Break-even future rate: *4.228%*

Break-even vs. current rate: *+149 bps*

Break-even % NPV savings: *6.23%*

Call option value as yield: *-0.517%*

etc.

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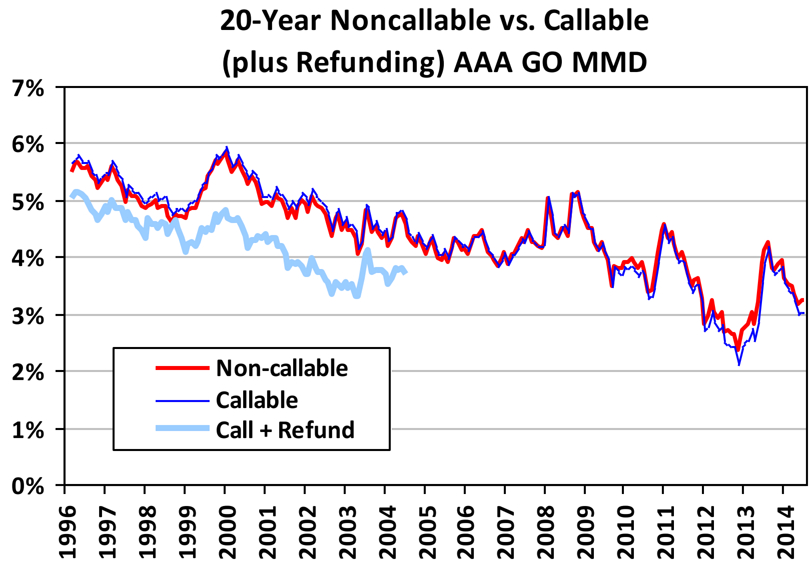
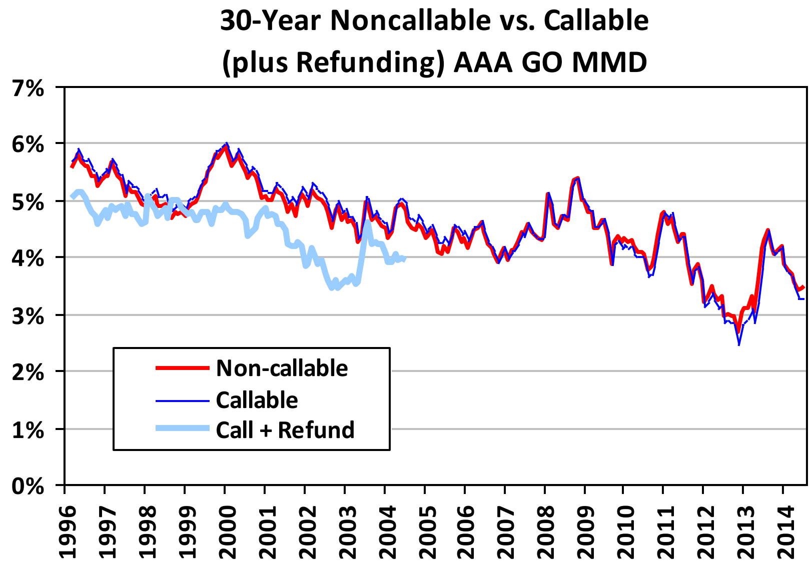
Historical Data on Performance of Callable vs. Non-callable Bonds

- Recent history shows non-callable bonds have underperformed versus callable bonds
- General trend of declining interest rates
- Maturity shift for replacement bonds, when “normal” yield curve has ascending slope





CABs and Convertible CABs

Economics and structures

intermediate bond math (Part 2)

Uses of CABs/Convertible CABs

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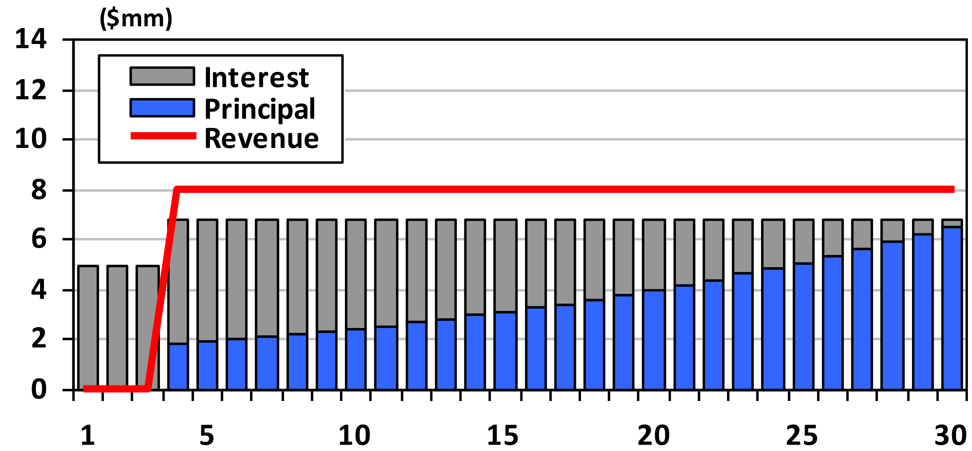
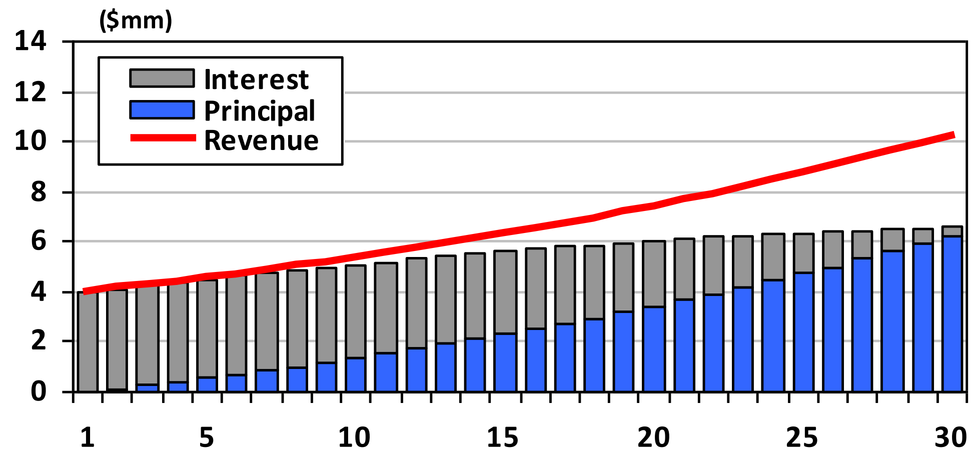
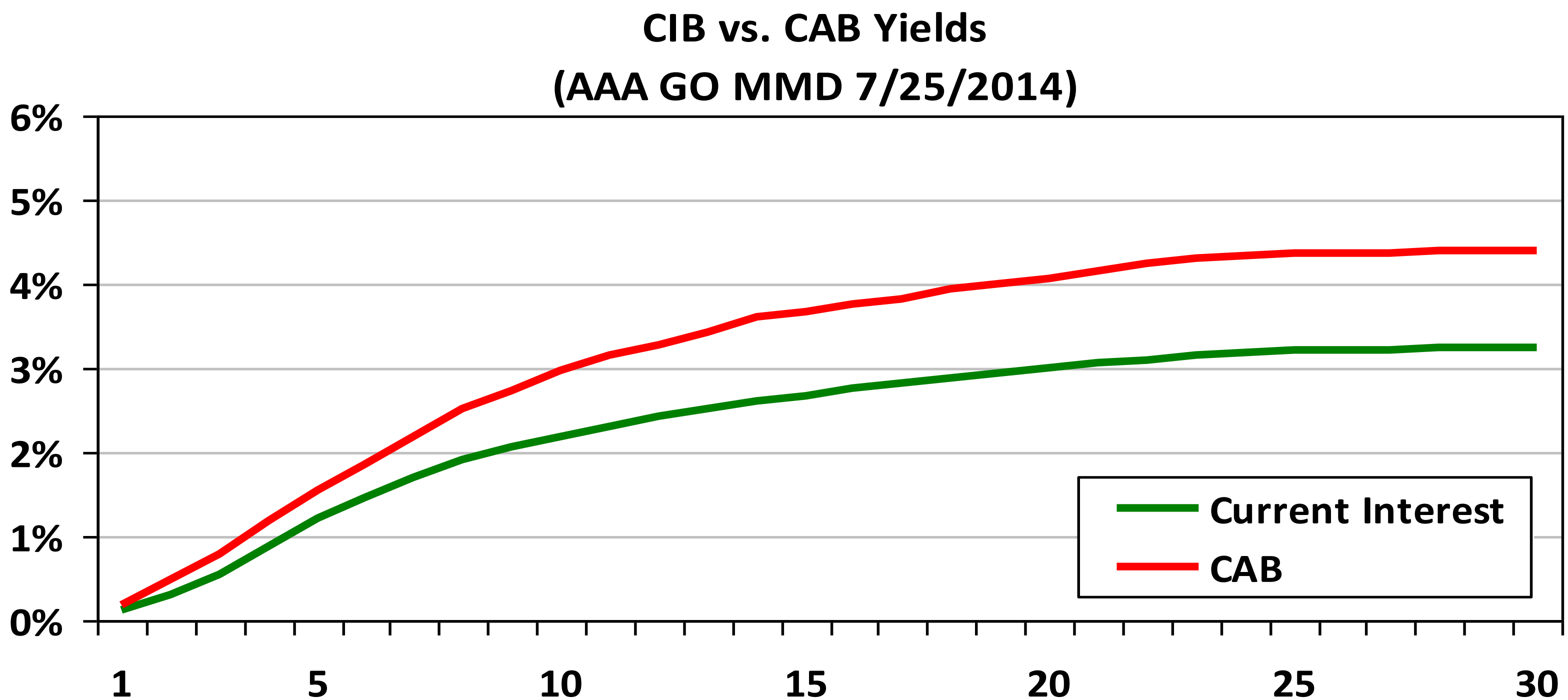
- Deferring principal reduces near-term debt service, but sometimes that is insufficient



- Revenue growth is projected to be steeply ascending (e.g., growth in volume and growth in price per unit volume), leaving untapped but needed bonding capacity



The Price of CABs/Convertible CABs



	Maturity	1	5	10	15	20	30
Current Interest	Yield	0.11%	1.21%	2.19%	2.68%	3.00%	3.24%
	Int./Prn.	0.11%	6.05%	21.90%	40.20%	60.00%	97.20%
CAB	Yield	0.19%	1.56%	2.97%	3.68%	4.05%	4.39%
	Int./Prn.	0.19%	8.08%	34.29%	72.80%	122.98%	267.94%

Imputing Zero-Coupon Bond Yields

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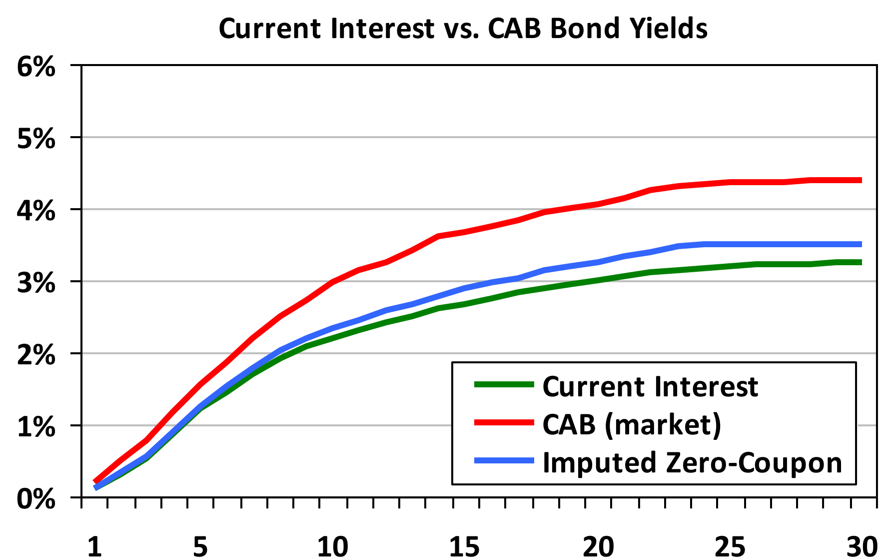
Year	Principal	Coupon	Yield	Interest	D/S	Price	Proceeds	Principal	Coupon	Yield	Interest	D/S	Price	Proceeds
1	830,000	2.00%	0.11%	169,500	999,500	101.888	845,670	870,000	2.00%	0.11%	128,050	998,050	101.888	886,426
2	845,000	3.00%	0.31%	152,900	997,900	105.359	890,284	885,000	3.00%	0.31%	110,650	995,650	105.359	932,427
3	870,000	4.00%	0.54%	127,550	997,550	110.282	959,453	915,000	4.00%	0.54%	84,100	999,100	110.282	1,009,080
4	905,000	5.00%	0.87%	92,750	997,750	116.201	1,051,619	950,000	5.00%	0.87%	47,500	997,500	116.201	1,103,910
5	950,000	5.00%	1.21%	47,500	997,500	118.334	1,124,173	1,000,000	0.00%	1.26%	0	1,000,000	93.926	939,257
Total	4,400,000			590,200	4,990,200		4,871,199	4,620,000			370,300	4,990,300		4,871,099
		1			2		3		1	4		2	3b	3

It is possible to calculate the theoretical yield of a CAB structure, based on current interest bond rates

1. Based on two structures that differ by either including or excluding CABs in the last maturity
2. Principal amortizations are solved to create equal debt service
3. / 3b. Proceeds of all CIBs are calculated, with the price of the CAB determined to result in equal total proceeds for the two structures
4. CAB's Yield can be calculated from the resulting price



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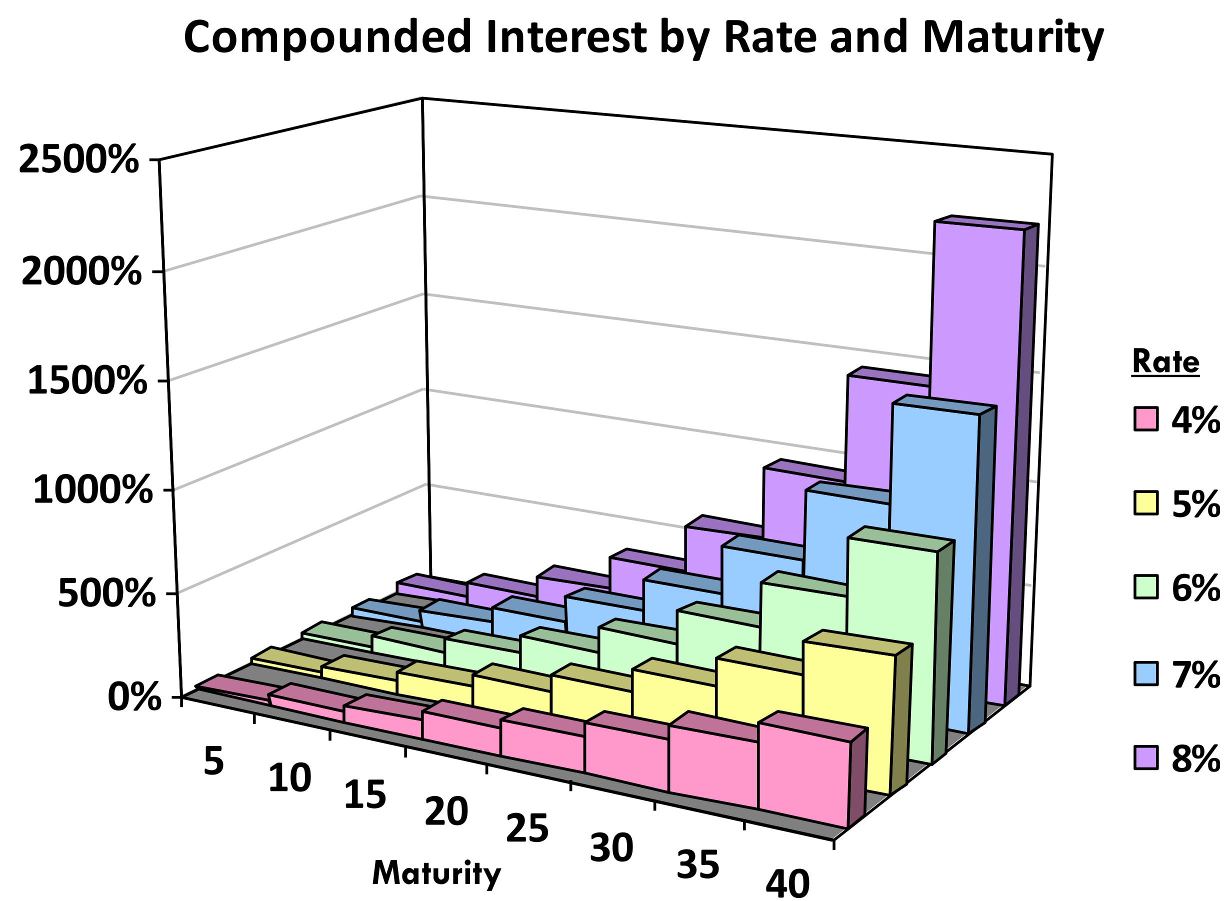
Compounded Rate of Interest

- CAB investors prefer long maturity structures
- Interest penalty also rise with longer maturities
- Additionally, CABs are generally non-callable making the commitment to pay interest irreversible



Bonus: Valuing Call Options

Economics and structures

intermediate bond math (Part 2)

Three Basic Approaches…

and Some Hybrid Approaches

Refunding Efficiency

What:

$$\frac{\text{NPV Savings}}{\text{(NPV Savings + Negative Arbitrage)}}$$

When:
Advance refunding

Why:
ALL rates can be known

Option Valuation Model

What:
Use models to project future interest rates, calculate savings and formulate as single PV value

When:
Consider multiple alternatives

Why:
Represents "market" perspective based on ability to hedge against future interest rates

Breakeven Analysis

What:
Find future interest rate at which refunding of two alternatives result in equivalent result

When:
Consider two alternatives, such as advance refunding and coupons

Why:
Results are easy to understand and rely very little on assumptions

Alternate Refunding Efficiency Calculation

What:

$$\frac{\text{NPV Savings}}{\text{Option Value}}$$

Evaluate Breakeven as Probability using Current Market Metrics

What:
Calculate probability that breakeven rate would be realized based on forward rates and volatilities

Refunding Efficiency Calculation

- Can follow same approach as estimating NPV savings (see slides 10 and 11)
  - ✓ Negative arbitrage is defined as the difference in escrow cost when investing at “new rate” versus at escrow yield

	A	B
1	Delivery	5/14/2014
2	Maturity	5/1/2020
3	Old Coupon	5.00%
4	New Rate	2.65%
5	Call Date	5/1/2015
6	Escrow Yield	0.25%
7	Call Price	100
8	COI	0.8%

Efficiency % =

(PRICE(B1,B2,B3,B4,100,2) –
PRICE(B1,B5,B3,B6,B7,2)/(1 – B9)) /

same

{(PRICE(B1,B2,B3,B4,100,2) –
PRICE(B1,B5,B3,B6,B7,2)/(1 – B9)) +

same

[PRICE(B1,B5,B3,B6,B7,2) –
PRICE(B1,B5,B4,B6,B7,2)]}

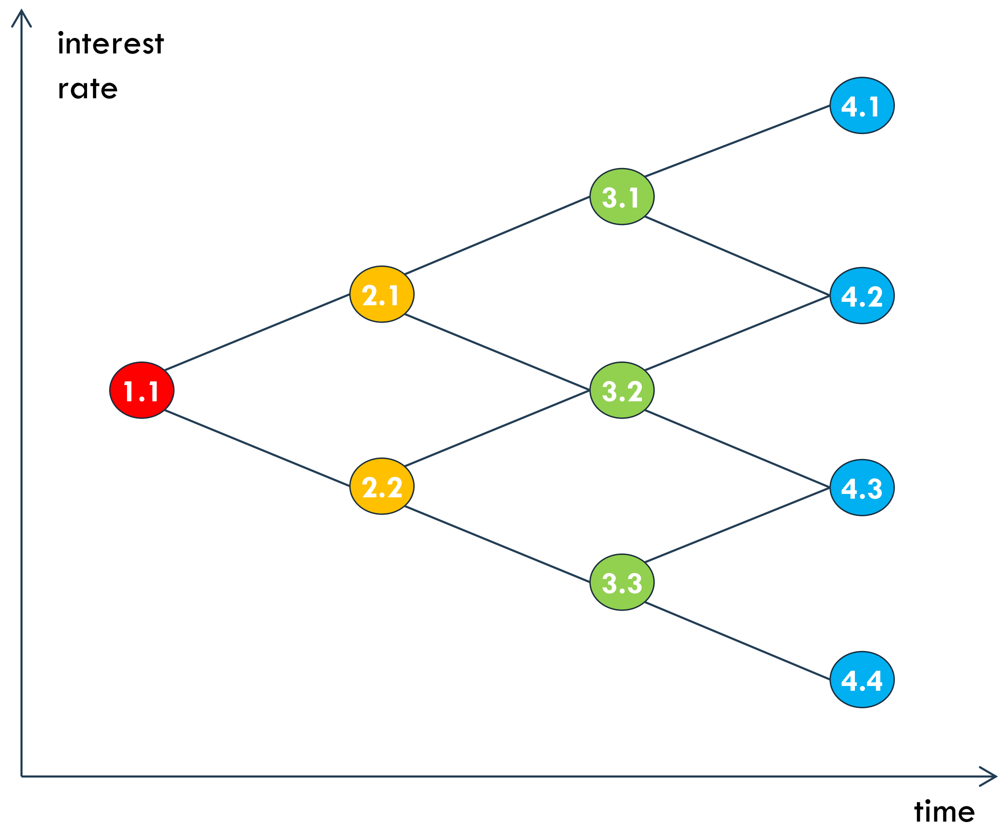
How Option Valuation Models Work

- A model generates future interest rates at different points in time
- NPV savings are calculated for each rate and at each time
- The value at each node is calculated as follows:

$$NPV^{1}_{m.n} = \max(NPV_{m.n}, \text{average}(NPV^{1}_{m+1.n}, NPV^{1}_{m+1.n+1}))$$

, where NPV is always >$0

- Option value is equal to $NPV^{1}_{1.1}$
- Results are <u>very</u> dependent on how interest rates are modeled



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Two Steps in a Breakeven Analysis

- Step 1: Find the future refunding interest rate (a.k.a., the breakeven rate) at which the economics of the two alternatives would be equivalent
- Steps 2: Determine whether or not the future rates would likely be above or below the breakeven right

Method	Compare vs. Current Rate	Compare vs. Interest Rate History	Assess Refunding Savings Level
How	Calculate difference between breakeven rate and current rate; is the amount of change likely?	Compare breakeven rate vs. historic distribution of interest rates; how often has rates been lower?	Calculate % NPV savings for breakeven refunding; is savings level realistic to achieve?
Why	Best for assessing near-term alternative; accuracy of interest rate outlook is more reliable	Appropriate for long-term alternative	Advance refunding would lock in savings early; chance can be measured using "personal" history
Example	Breakeven rate is +150 bps from current over 6 months	Breakeven rate is in 80^{th} percentile	Breakeven NPV savings is 2.78%

Questions?

Thank you for your participation!